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Post-buckling analysis of composite plates containing embedded delaminations with arbitrary shape by using higher order shear deformation theory

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ABSTRACT

The compressive post-buckling behavior of composite laminates containing embedded delamination with arbitrary shape is investigated analytically. For modeling the embedded delamination, the laminate is divided into three smaller regions. The higher order shear deformation theory is implemented and the formulation is based on the Rayleigh–Ritz approximation technique by the application of the simple/complete polynomial series for each region. The nonlinear equilibrium equations, which are achieved through the application of the principle of Minimum Potential Energy, are solved by employing the Newton–Raphson iterative procedure. Some interesting results are obtained and compared with those achieved by the finite element method of analysis using ANSYS commercial software. A good agreement is seen to exist between the results. This is while for a given level of accuracy in the results, ANSYS requires a markedly larger number of degrees of freedom compared to that needed by the developed method. Moreover, a considerable reduction in the load carrying capacity of laminate is noticed due to the presence of delamination.

1. Introduction

Delamination has been a subject of major concern in engineering applications of composite laminates because of the associated problems of structural stability, reduction in load-bearing capacity, stiffness degradation and fracture. Many delamination related studies have primarily focused on the prediction of the buckling load and post-buckling behavior. This is because the presence of the delamination causes reductions in the bending stiffness which in turn leads to the undesirable loss in the compressive buckling and post-buckling strength. Various methods have been proposed for the analysis of a plate that contains through-the-width and embedded delaminations. Chai et al. have established an analytical one-dimensional model for the analysis of delamination buckling of beam-plate in 1981 [1]. Shivakumar et al. have studied the buckling behavior of thin elliptical delamination using the Rayleigh–Ritz and finite element method [2]. Kardomateas has focused on the post-buckling behavior of thin delaminations in delaminated Kevlar/epoxy laminates under large applied displacements and reported some experimental results on the macroscopic behavior [3]. Piao has used a consistent shear deformation theory to analyze the beam-plate delamination buckling [4]. Nilsson et al. have performed an experimental investigation of buckling induced delamination growth [5]. Adan et al. have solved the governing differential equation for beams with multiple through the width delaminations to find the buckling load [6,7].

Gu and Chattopadhyay have used higher order shear deformation theory to study the buckling behavior of delaminated composite plates [8]. Shahwan and Wass have used the nonlinear spring distribution between a thin plate which is bonded laterally to a thick plate to analyze the buckling problem [9]. Suemasu et al. have studied the effects of circular and multiple delaminations on the compressive buckling and failure load. They have observed that although the failure strength depends on the toughness of the matrix resin, the buckling loads are unaffected by the toughness of the resin. However, these experiments either have focused on the critical load or are performed by using very thin or unidirectional composite specimens [10]. Jane et al. have analyzed the post local buckling behavior of laminated rectangular plates by implementing Rayleigh–Ritz method and using Von Karman's nonlinear strain displacement relations [11]. Andrews et al. have formulated a technique by utilizing the classical laminated plate theory to study the elastic interaction of the multiple through the width delaminations in laminated plates subject to static out of plane loading while deforming in cylindrical bending [12]. Kharazi et al. have investigated the buckling of composite laminates with a through the width delamination by using different plates theories. Their method is based on Rayleigh–Ritz approximation technique [13] and it

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can handle both local buckling of the delaminated sublaminate and global buckling of the whole plate. Kharazi et al. have also investigated the compressional stability behavior of composite plates with multiple through-the-width delaminations by using higher order shear deformation theory [14]. Ovesy et al. have investigated post-buckling analysis of composite plates containing embedded delaminations by using higher order shear deformation theory [15]. They have analyzed plates with square or circle delaminated regions and their formulation are developed specifically for these shapes of delaminations. In their work, which is based on the Rayleigh–Ritz approximation technique, the shape of the delaminated region has an important role in defining the corresponding shape functions. Thus, the shape of the delaminated regions are usually considered to be of simple types such as circles or squares in order to avoid the difficulties in satisfying continuity conditions that might occur with respect to complex shapes. In the current paper, however, the compressive post-buckling behavior of composite laminates containing embedded delamination with arbitrary shape is investigated analytically. The analytical method is based on the higher order shear deformation theory and its formulation is developed on the basis of the Rayleigh–Ritz approximation technique by the implementation of the simple and complete polynomial series. Although the presented method can be employed for the buckling as well as the post-buckling analysis of the delaminated plates, the focus of this paper is on the post-buckling behavior of the plates. Some interesting results are obtained and compared with those achieved by the finite element method of analysis using ANSYS commercial software. It will be seen that for a given level of accuracy in the results, ANSYS requires a markedly larger number of degrees of freedom compared to that needed by the developed method.

2. Modeling of the arbitrary shape embedded delamination

In this section the analytical model and the applied theory in this study are briefly outlined. The Reddy's third order shear deformation theory is applied in the analytical formulation. Thus the assumptions of this theory are:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) - z^3 C_1\left(\phi_x + \frac{\partial w_0}{\partial x}\right) \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) - z^3 C_1\left(\phi_y + \frac{\partial w_0}{\partial y}\right) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u,v and w are components of displacements at a general point, whilst u_0,v_0 and w_0 are similar components at the middle surface ($Z = 0$). In addition ϕ_x and ϕ_y are the rotations of the mid-plane normals about y and x axis respectively. Besides $C_1 = \frac{4}{3h^2}$ where h is the thickness of the whole laminate. Using Eq. (1) in the Green's expression for nonlinear strains and neglecting lower order terms in a manner consistent with the usual Von Karman assumption gives the following expressions for strain at a general point:

$$\begin{aligned} \{\bar{\varepsilon}\} &= \{\varepsilon^0\} + z\{\varepsilon^1\} + z^3\{\varepsilon^3\} \\ \{\bar{\gamma}\} &= \{\gamma^0\} + z^2(\gamma^2) \end{aligned} \tag{2a}$$

where $\bar{\varepsilon}_{ij}$ and $\bar{\gamma}_{ij}$ are different components of the strain tensor.

$$\begin{aligned} \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\gamma}_{xy} \end{Bmatrix} &= \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z\begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} + z^3\begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix} \\ \begin{Bmatrix} \bar{\gamma}_{yz} \\ \bar{\gamma}_{xz} \end{Bmatrix} &= \begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} + z^2\begin{Bmatrix} \gamma^0_{yz} \\ \gamma^0_{xz} \end{Bmatrix} \end{aligned} \tag{2b}$$

and

$$\begin{aligned} &\begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix} \\ &\begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix} = -C_1 \begin{Bmatrix} \left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) \\ \left(\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right) \\ \left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial y \partial x}\right) \end{Bmatrix} \\ &\begin{Bmatrix} \gamma^{(0)}_{yz} \\ \gamma^{(0)}_{xz} \end{Bmatrix} = \begin{Bmatrix} \phi_y + \frac{\partial w_0}{\partial y} \\ \phi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma^{(2)}_{yz} \\ \gamma^{(2)}_{xz} \end{Bmatrix} = -C_2 \begin{Bmatrix} \left(\phi_y + \frac{\partial w_0}{\partial y}\right) \\ \left(\phi_x + \frac{\partial w_0}{\partial x}\right) \end{Bmatrix} \end{aligned} \tag{2c}$$

where $C_2 = 3C_1$. It is noted that the C_1 parameter is defined just for the higher order deformation theory and it results in $\boldsymbol{\varepsilon}^3$ and $\boldsymbol{\gamma}^2$ terms, which are vanished in the general expression of the strain in the classical and first order shear deformation theories.

The stress–strain relationship at a general point for the plates becomes:

$$\begin{Bmatrix} \bar{\sigma}_{xx} \\ \bar{\sigma}_{yy} \\ \bar{\tau}_{xz} \\ \bar{\tau}_{yz} \\ \bar{\tau}_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & 0 & 0 & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & 0 & 0 & \overline{Q}_{26} \\ 0 & 0 & \overline{Q}_{44} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55} & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & 0 & 0 & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\gamma}_{xz} \\ \bar{\gamma}_{yz} \\ \bar{\varepsilon}_{xy} \end{Bmatrix} \tag{3}$$

where $\overline{Q}_{ij}$ are the transformed reduced stiffness coefficients. The constitutive equations for a plate can be obtained through the use of Eqs. (2) and (3) and appropriate integration through the thickness.

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} + z^3 \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix} \right) dz \tag{4a}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z^2 \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} + z^4 \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix} \right) dz$$

$$\begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \left(z^3 \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} + z^4 \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} + z^6 \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^3_{xx} \\ \varepsilon^3_{yy} \\ \gamma^3_{xy} \end{Bmatrix} \right)$$