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Buckling and free vibration finite strip analysis of composite plates with cutout based on two different modeling approaches

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ABSTRACT

A Reddy type, third order shear deformation theory of plates is applied to the development of two versions of finite strip method (FSM), namely semi-analytical and spline methods, to predict the behavior of the moderately thick plates containing cutouts. The internal cutouts are modeled based on two different modeling approaches, and the effects of cutouts on the buckling critical stresses as well as natural frequencies are investigated.

1. Introduction

Plate structures are considered as major engineering substructures, especially wherever the weight is a main issue. In aerospace and marine structures, the structural weight beside the necessary strength is of chief importance. Moreover, good strength-to-weight and stiffness-to-weight ratios as well as design versatility have made composite laminates very popular engineering structures. In some engineering structures, the panels may be perforated to allow for cabling and piping. These structures may be subjected to in-plane compressive loading, in which case the "buckling" phenomenon becomes a critical design criterion. Also, the natural frequencies of every structural element are of great significance in order to avoid the so-called "resonance" conditions.

Cheung and Kong [1] developed a modified spline finite strip element and applied it to the stability analysis of isotropic and laminated plates with square cut-outs under constant uniform initial stresses. Anil et al. [2] utilized FEM in order to investigate the cutout effects on the buckling strength of moderately to very thick composite laminated plates under in-plane compressive loads. Eccher et al. [3] introduced and applied the isoparametric spline finite strip method to investigate the elastic buckling analysis of perforated folded thin-walled plate structures. The method was developed on the basis of constitutive relations of the Mindlin plate theory. Recently, the authors of the current paper have developed a semi-analytical finite strip (S-a FSM) formulation as well as a B-spline finite strip (spline FSM) formulation on the basis of classical shells assumptions and subsequently investigated the parametric instability of layered composite flat and cylindrical thin-walled structures [4]. In applying classical shell assumptions, the formulation ignores the effects of transverse through the thickness strains. This means that the corresponding analysis is not sensitive to the variation of the thickness of the structure. On the other hand, a higher order through the thickness approximation of the strains takes a due account of the variation of the thickness. Thus, the authors of the current paper have developed FSM based on the Reddy's higher order shells theory to investigate the parametric instability of moderately thick structures [5,6].

In the present paper, two versions of FSM, namely the S-a FSM and spline FSM are developed and adapted to deal with the plate models containing cutouts. The formulations are based on Reddy type higher order plates theory in order to include the transverse shear stresses effect in case of moderately thick plates. Two approaches are developed and applied to the FSM formulations in order to model the cutout effects. The advantages and disadvantages of each approach are investigated. For example, one of the aforementioned approaches, whilst being more accurate, is only applicable to the rectangular cutout shape and can only be implemented by the spline FSM formulation. However, at the expense of some accuracy, the other approach has the advantages of having fewer degrees of freedom, having the ability to deal with different cutout shapes as well as being suitable for both spline and S-a FSM formulations. To the best of author's knowledge, the latter modeling approach has not been utilized in FSM formulation, previously.

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2. Theoretical developments

The structure geometry is assumed to be made up of a series of longitudinal flat strips. Fig. 1 depicts a general flat finite strip with length L, width b_s and total thickness t. The strip may be subjected to uniform static in-surface loadings.

In order to include the effects of through the thickness transverse shear strains as parabolic variation functions of the thickness dimension, a third order algebraic interpolation is assigned for the displacement field approximation [5,6]. Applying the stress-free conditions at the two free surfaces of the flat strip, the displacement functions may be simplified as,

$$\begin{cases} u(x,y,z;t) = u^o + \beta_x\left(z + \frac{-4}{3t^2}z^3\right) + \frac{\partial w^o}{\partial x}\left(\frac{-4}{3t^2}z^3\right) \\ v(x,y,z;t) = v^o - \beta_y\left(z + \frac{4}{3t^2}z^3\right) - \frac{\partial w^o}{\partial y}\left(\frac{-4}{3t^2}z^3\right) \\ w(x,y,z;t) = w^o \end{cases} \tag{1}$$

where u, v, w are the arbitrary point displacements whilst u^o, v^o, w^o are related to displacements of the corresponding coordinates on the middle surface. There are five unknown functions in the right-hand side of the approximation equations (i.e. $u^o, v^o, w^o, \beta_x, \beta_y$) that are to be defined.

The linear strains are expressed as,

$$\begin{aligned} &\varepsilon_x = u_{,x}, \quad \varepsilon_y = v_{,y}, \quad \gamma_{xy} = u_{,y} + v_{,x} \\ &\gamma_{yz} = v_{,z} + w_{,y}, \quad \gamma_{xz} = u_{,z} + w_{,x} \end{aligned} \tag{2}$$

Substituting the flat strip displacement functions Eq. (1) into the linear strain terms Eq. (2) leads to the strain field as, [5,6]

$$\begin{cases} \varepsilon_x = \varepsilon_x^{(0)} + z\cdot\varepsilon_x^{(1)} + z^2\cdot\varepsilon_x^{(2)} + z^3\cdot\varepsilon_x^{(3)} \\ \varepsilon_y = \varepsilon_y^{(0)} + z\cdot\varepsilon_y^{(1)} + z^2\cdot\varepsilon_y^{(2)} + z^3\cdot\varepsilon_y^{(3)} \\ \gamma_{xy} = \gamma_{xy}^{(0)} + z\cdot\gamma_{xy}^{(1)} + z^2\cdot\gamma_{xy}^{(2)} + z^3\cdot\gamma_{xy}^{(3)} \end{cases} \quad \begin{cases} \gamma_{yz} = \gamma_{yz}^{(0)} + z\cdot\gamma_{yz}^{(1)} + z^2\cdot\gamma_{yz}^{(2)} + z^3\cdot\gamma_{yz}^{(3)} \\ \gamma_{xz} = \gamma_{xz}^{(0)} + z\cdot\gamma_{xz}^{(1)} + z^2\cdot\gamma_{xz}^{(2)} + z^3\cdot\gamma_{xz}^{(3)} \end{cases} \tag{3}$$

where the strain coefficients are:

$$\begin{cases} (\varepsilon_x^{(0)}, \varepsilon_y^{(0)}, \gamma_{xy}^{(0)}) = (u_{,x}^o, v_{,y}^o, u_{,y}^o + v_{,x}^o) \\ (\varepsilon_x^{(1)}, \varepsilon_y^{(1)}, \gamma_{xy}^{(1)}) = (\beta_{x,x}, -\beta_{y,y}, \beta_{x,y} - \beta_{y,x}) \\ (\varepsilon_x^{(2)}, \varepsilon_y^{(2)}, \gamma_{xy}^{(2)}) = (0,0,0) \\ (\varepsilon_x^{(3)}, \varepsilon_y^{(3)}, \gamma_{xy}^{(3)}) = (\beta_{x,x}^*, -\beta_{y,y}^*, \beta_{x,y}^* - \beta_{y,x}^*) \end{cases} \quad \begin{cases} \left(\gamma_{yz}^{(0)}, \gamma_{xz}^{(0)}\right) = (w_{,y}^o - \beta_y, w_{,x}^o + \beta_x) \\ \left(\gamma_{yz}^{(1)}, \gamma_{xz}^{(1)}\right) = (0,0) \\ \left(\gamma_{yz}^{(2)}, \gamma_{xz}^{(2)}\right) = (-3\beta_y^*, 3\beta_x^*) \\ \left(\gamma_{yz}^{(3)}, \gamma_{xz}^{(3)}\right) = (0,0) \end{cases}$$

$$\beta_x^* = C_1(w_{,x} + \beta_x), \beta_y^* = C_1(w_{,y} - \beta_y), C_1 = -4/3t^2 \tag{4}$$

A laminated strip is considered to be made from some orthotropic property laminas. The force resultants, N, M, P, Q, T, could be expressed as [5,6],

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{P\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] & [E] \\ [B] & [D] & [F] \\ [E] & [F] & [H] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^{(0)}\} \\ \{\varepsilon^{(1)}\} \\ \{\varepsilon^{(3)}\} \end{Bmatrix}, \quad \begin{Bmatrix} \{Q\} \\ \{T\} \end{Bmatrix} = \begin{bmatrix} [A^S] & [D^S] \\ [D^S] & [F^S] \end{bmatrix} \begin{Bmatrix} \{\gamma^{(0)}\} \\ \{\gamma^{(2)}\} \end{Bmatrix} \tag{5}$$

Solution of the problem is sought through the principle of virtual work. Total energy is summation of kinetic (T), pre-stress (U_g) and strain ($U_{elastic}$) energy components,

$$\Pi = U_{elastic} - U_g - T \tag{6}$$

$$\begin{aligned} &T = \frac{1}{2}\rho t \int\int_L \left(\dot{u}^{o2} + \dot{v}^{o2} + \dot{w}^{o2} + t^2\left(\dot{\beta}_x^2 + \dot{\beta}_y^2\right)\Big/12\right)dx\,dy \\ &U_g = \frac{1}{2}t\int\int_L \left(N_x\left[u_{,x}^{o2} + v_{,x}^{o2} + w_{,x}^{o2} + t^2\left(\beta_{x,x}^2 + \beta_{y,x}^2\right)\Big/12\right]\right)dx\,dy \\ &U_{elastic} = \frac{1}{2}\int\int_L \left(\langle N\,M\,P\rangle\cdot\langle\varepsilon^{(0)}\,\varepsilon^{(1)},\varepsilon^{(3)}\rangle^T + \langle Q,T\rangle\cdot\langle\gamma^{(0)}\,\gamma^{(2)}\rangle^T\right)dx\,dy \end{aligned} \tag{7}$$

where ρ is the uniform material mass density, '.' represents the differentiation with respect to time and ',' defines a non-time differentiation operator. Substituting the energy terms Eq. (7) in Eq. (6), applying the principle of virtual work, factorizing with respect to the degrees of freedom vectors to build the matrices, assembling the strip matrices and implementing the necessary boundary conditions, a system of equations for the model is obtained as follows,

$$M\ddot{\delta} + (K - K_g)\,\delta = 0 \tag{8}$$

where M, K and K_g are the square global structural matrices corresponding to mass, strain energy and initial stress energies, respectively. Whenever the initial stress is ignored, Eq. (8) reduces to a free vibration problem whilst in case of static assumptions, with the presence of initial stress, the problem changes to a static buckling problem. Both of problems are eigen-value type problems as follows:

$$\begin{aligned} &(\text{free vibration problem}) \quad M\ddot{\delta} + K\delta = 0 \rightarrow |K - \omega^2 M| = 0 \\ &(\text{buckling problem}) \quad K\delta - K_g\,\delta = 0 \rightarrow |K - \lambda[K_g]| = 0 \end{aligned} \tag{9}$$

By utilizing the QR method to solve these eigen-value problems, the free vibration frequencies and also the buckling critical stresses could be obtained.

The mid-surface displacements and rotations are the same unknown functions designated in the right-hand sides of Eq. (1). In case of semi-analytical version of FSM, the trigonometric functions are assumed as the longitudinal approximation functions while a series of Bezier-spline of third order are utilized in the same direction for the spline version. The transverse approximation functions are polynomials of various types and orders. The undetermined displacement coefficients are related to transverse terms in semi-analytical version but are coefficients of longitudinal terms in the spline version. The linear Lagrange polynomials are used in

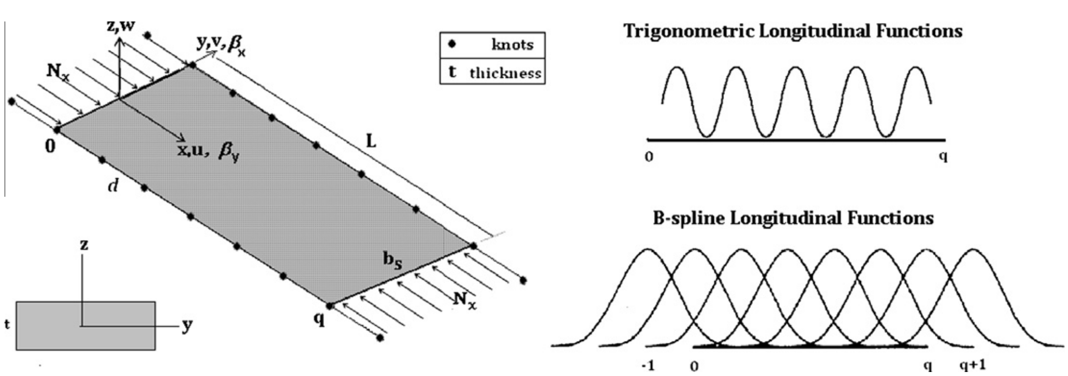

Fig. 1. Geometry, loading configuration and longitudinal estimation functions for a typical flat strip.