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Homogenization of metallic fiber-reinforced composites under stochastic ageing

M. Kamiński [a,b,*], M. Leśniak [a]

[a] Department of Steel Structures, Technical University of Łódź, Faculty of Civil Engineering, Architecture and Environmental Engineering, Al. Politechniki 6, 90-924 Łódź, Poland
[b] Institute of Polymer Research, Hohe Strasse 6, D-01069 Dresden, Germany

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ABSTRACT

The main aim of this paper is to present an application of the generalized stochastic perturbation technique to model stochastic ageing processes of the metallic fibre-reinforced periodic composite materials in terms of their effective properties. Those ageing processes are modelled here as two-parametric time series having Gaussian random initial values and time rate, both defined uniquely by their expectations and standard deviations. Computational homogenization procedure is discrete and based on the Finite Element Method program MCCEFF as well as the computer algebra system MAPLE, where the Response Function Method and the stochastic analysis are entirely implemented. This numerical strategy is used to analyze probabilistic moments of the effective elastic tensor of the few metal matrix composites as well as to simulate stochastic ageing of two representative composites – $MoSio_2$–SiC and Ti–SiC. The approach proposed and results of computations may be further applied in the reliability analysis of metallic or the other composites.

1. Introduction

A computational modeling of the stochastic processes based on the Monte-Carlo simulation method is known for their large time consumption because the entire generalizations of random populations in different discrete time moments are evaluated and used in various Finite Element Method models. More optimal strategy would be based on the observation of the probabilistic moments of the examined processes in the same time moments as before performed with at least comparable accuracy. The generalized stochastic perturbation technique based on the Taylor expansion of the uncertain parameters and the state functions may be useful in this case, to determine for instance the probabilistic moments of the effective elasticity tensor. However, we need to have the analytical description for the basic moments of the stochastic processes being modeled to assure the sufficient input for the perturbation-based analysis. This process does not need to have the Gaussian realizations for the whole history, however the lack of correlation between various parameters simplifies significantly the analysis. We can use in this purpose some forms of the ageing processes known from computational biology [1] (as the exponential forms), power-laws popular in various branches of engineering [2] or just the linear decay [4] with random coefficients popular in the civil engineering inspections.

Contrary to the second order second moment implementations of the perturbation technique, the hierarchical equations are not solved here for the increasing order approximants for the probabilistic output. Now, the Response Function Method (RSM) is explored, where the polynomial interrelation between the stochastic output and input is to be approximated symbolically via several deterministic solutions around the mean values of the stochastic parameters in various time moments of the process. Finally, one can obtain a discrete polynomial approximation of the stochastic process as the function of the initial stochastic process of the ageing, for instance. The method is similar to the Response Surface Method known from the literature [9], but instead of the polynomial form of the lower order for multiple parameters we use here higher order approximation for a single variable. It should be underlined that the use of multiple variables is also allowable, however we would like to distinguish between the influence of different physical quantities influencing the time fluctuations of the effective tensor probabilistic characteristics. We need to emphasize that the classical Finite Element Method [11] programs (with and without the access to the solver source code) may be employed and extended using the proposed approach.

The engineering practice with composites (and even classical homogeneous engineering materials) shows that the ageing of materials (neglecting the real nature of this mechanism) is dangerous for many structures and elements and should be included into the designing process. An integral part of such a designing process should be mathematical equation simulating the ageing behavior of various structural elements and materials, a proper computational modeling technique as well as the final conclusion stating the safe time of operation for the specific engineering structure in the given environment. The mathematical equation responsible for the ageing process may be proposed using the strength verification, however it would need the very large number of

* Corresponding author at: Department of Steel Structures, Technical University of Łódź, Faculty of Civil Engineering, Architecture and Environmental Engineering, Al. Politechniki 6, 90-924 Łódź, Poland.
E-mail address: Marcin.Kaminski@p.lodz.pl (M. Kamiński).

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experimentation extended for the very long periods of time. Instead of it, one may adopt some ageing model proposed extensively in the literature and following the research made in various applied science and engineering branches – like exponential or linear ageing decay for different physical or mechanical parameters.

Taking into account the above considerations, the stochastic ageing laws applied for the Young moduli of the composite components are examined in terms of the effective parameters for the periodic fiber-reinforced composites with metallic components. The linear decay responsible for the ageing process is adopted, where the initial value and the ageing velocity are the Gaussian random parameters having first two moments constant in time. This definition of the stochastic process enables to determine the basic probabilistic moments for the considered parameters and to include them into the homogenization procedure engaged here to analyze the metallic composites. Next this procedure is based on the Finite Element Method program MCCEFF determining the homogenized stochastic elasticity tensor in plane strain (for the rectangular periodicity cell and the fiber embedded into it) using the generalized stochastic perturbation technique. Some additional algebraic computations are completed in the system MAPLE (interoperating with the program MCCEFF), where the response functions are found and used to finally determine stochastic processes and where graphical representations for the stochastic processes are provided. Let us underline that the comparison of the tenth order perturbation approach with the Monte-Carlo simulation results were proven before in various computational experiments with random variables [3–5], so that it is employed here to discuss the influence of probabilistic and stochastic fluctuations in Young moduli of metallic composites on the probabilistic moments for their composite effective parameters.

2. Fibre-reinforced composite model

The periodic fiber-reinforced composite structure in the plane strain with linear elastic and transversely isotropic components and some stochastic parameters is considered now. Let us denote the Representative Volume Element (RVE) of this composite as Ω; $Y \subset \Re^2$ stands for the section of this composite with $x_3 = 0$ plane being constant along the x_3 axis parallel to the fibers direction (Fig. 1).

Let the region Ω contain two perfectly bonded, coherent and disjoint subsets Ω_1 (fiber) and Ω_2 (matrix) and let the scale between corresponding geometrical diameters of Ω and Y is described by some small and real parameter $\varepsilon > 0$. Let $\partial\Omega$ denote external boundary of the Ω while $\partial\Omega_{12}$ – the interface boundary between Ω_1 and Ω_2 regions. The elasticity tensor is defined here as

$$\begin{aligned} C_{ijkl}(\mathbf{x}) &= B_{ijkl}(\mathbf{x})\, e(\mathbf{x}) \\ &= e(\mathbf{x})\left\{\delta_{ij}\delta_{kl}\frac{\nu(\mathbf{x})}{(1+\nu(\mathbf{x}))(1-2\nu(\mathbf{x}))}\right. \\ &\quad \left. +(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\frac{1}{2(1+\nu(\mathbf{x}))}\right\} \end{aligned} \quad (1)$$

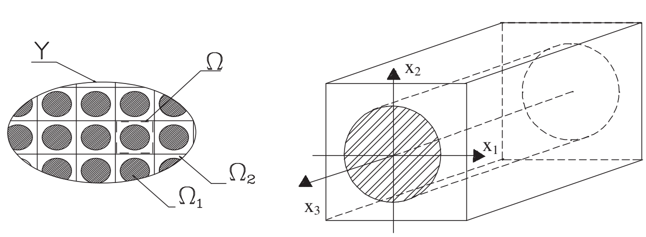

Fig. 1. Periodic fiber reinforced composite.

The effective tensor $C_{ijkl}^{(eff)}$ (for the artificial homogenized composite) is introduced as such a tensor that replacing C_{ijkl}^{ε} (for the real composite) with $C_{ijkl}^{(eff)}$ in the following equilibrium equations:

$$C_{ijkl}^{\varepsilon}\varepsilon_{kl}(\mathbf{u}^{\varepsilon}) + f_i = 0;\ \mathbf{x} \in \Omega \quad (2)$$

$$\varepsilon_{ij}(\mathbf{u}^{\varepsilon}) = \frac{1}{2}(u_{i,j}^{\varepsilon} + u_{j,i}^{\varepsilon});\ \mathbf{x} \in \Omega \quad (3)$$

$$C_{ijkl}^{\varepsilon}(\mathbf{x}) = \chi_1(\mathbf{x})C_{ijkl}^{(1)} + (1-\chi_1(\mathbf{x}))C_{ijkl}^{(2)} \quad (4)$$

where $\mathbf{u}^0$ is obtained as a solution being a weak limit of $\mathbf{u}^{\varepsilon}$ with $\varepsilon \to 0$ and where the characteristic function defining the elastic parameters equals to

$$\chi_1(\mathbf{x}) = \begin{cases} 1, & \mathbf{x} \in \Omega_1 \\ 0, & \mathbf{x} \in \Omega_2 \end{cases} \quad (5)$$

with the boundary conditions

$$\mathbf{u}^{\varepsilon} = 0;\ \mathbf{x} \in \partial\Omega. \quad (6)$$

Now, let us consider the stochastic variations in Young modulus within the composite, i.e. [4]

$$C_{ijkl}(\mathbf{x};\ \omega;\ t) = B_{ijkl}(\mathbf{x})e(\mathbf{x};\ \omega;\ t), \quad (7)$$

with the following representation:

$$e(\mathbf{x};\ \omega;\ t) = -\dot{e}(\mathbf{x};\ \omega)t + e^0(\mathbf{x};\ \omega). \quad (8)$$

The random field $e^0(\mathbf{x};\omega)$ is equivalent to the initial Young moduli of the composite constituents, whereas $\dot{e}(\mathbf{x};\ \omega)$ represents the velocity of ageing process for the matrix and the fibre separately, i.e.

$$\dot{e}(\mathbf{x};\ \omega) = \chi_1(\mathbf{x})\dot{e}_1(\omega) + (1-\chi_1(\mathbf{x}))\dot{e}_2(\omega). \quad (9)$$

It is assumed that this process is of course continuous in time and the particular components are fully uncorrelated from each other (Young modulus of both components). Taking into account the relation (5) one can write that

$$E[\dot{e}(x;\omega)] = \chi_1 E[\dot{e}_1] + (1-\chi_1)E[\dot{e}_2] \text{ for } \chi_1 = \begin{cases} 1 & x \in \Omega_1, \\ 0 & elsewhere, \end{cases} \quad (10)$$

the variance is defined accordingly as

$$Var(\dot{e}(x;\omega)) = \chi_1 Var(\dot{e}_1) + (1-\chi_1)Var(\dot{e}_2). \quad (11)$$

3. Homogenization method

Problem: Determine the series of probabilistic moments $\mu_m\left(C_{ijkl}^{(eff)}\right)$ for Ω using the lemma

$$C_{ijkl}^{(eff)} = \frac{1}{|\Omega|}\int_{\Omega}(C_{ijkl}(\mathbf{y}) + C_{ijmn}(\mathbf{y})\varepsilon_{mn}(\chi_{(kl)}(\mathbf{y})))d\Omega \quad (12)$$

with periodic and kinematically admissible homogenization function $\chi_{(ij)k}$ being a solution to the local problem on Y:

$$a_y((\chi_{(ij)k} + y_j\delta_{ki})\mathbf{n}_k, \mathbf{w}) = 0, \quad (13)$$

for any periodic $\mathbf{w}$ ($\mathbf{n}_k$ is the unit coordinate vector). A bilinear form $a^{\varepsilon}(\mathbf{u},\mathbf{v})$

$$a^{\varepsilon}(\mathbf{u},\mathbf{v}) = \int_{\Omega} C_{ijkl}\left(\frac{\mathbf{x}}{\varepsilon}\right)\varepsilon_{ij}(\mathbf{u})\varepsilon_{kl}(\mathbf{v})d\Omega, \quad (14)$$

together with the linear one (including body forces and von Neumann boundary conditions)

$$L(\mathbf{v}) = \int_{\Omega} f_i v_i\, d\Omega + \int_{\partial\Omega_{\sigma}} p_i v_i\, d(\partial\Omega). \quad (15)$$