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Buckling of composite thin walled beams by refined theory

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ABSTRACT

This work presents the buckling analysis of laminated composite thin walled structures by the 1D finite element based unified higher-order models obtained within the framework of the Carrera Unified Formulation (CUF). In the present study, the refined beam theories are obtained on the basis of Taylor-type expansions. The finite element analysis has been chosen to easily handle arbitrary geometries as well as boundary conditions. Buckling behavior of laminated composite beam and flat panels are analyzed to illustrate the efficacy of the present formulation and various types of buckling modes are observed depending on the geometrical and material parameters. It is observed that the lower order models are unable to deal with torsion.

1. Introduction

Composite laminated structures are increasingly being used in the design of load-carrying members for the aerospace, civil and modern engineering applications. These composites are often very susceptible to buckle in various modes. Moreover, the classical theories have been shown to underpredict the deflections and to overpredict the buckling loads [1,2] of these structures. Since much emphasis is given in achieving an optimized ratio of the structure's weight to strength ratio in modern structures, the accurate prediction of their stability limit state is of fundamental importance in the design of thin-walled laminated composite structures. Further, for the case of flat panels, the torsional modes usually occur very close to the critical buckling loads so, use to refined higher order theory which accounts for out of plane displacements is also important.

The first available work in open literature on flexural torsional buckling seems to due to Michell [3] and Prandtl [4] on solid rectangular beams. Significant work on homogeneous beams, based on thin tube theory assumptions, has been carried out by Timoshenko and Gere [5], Hodges and Peters [6], Reissner [7], Hodges [8]. It is apt to make a mention here that [7] was the first one who incorporated the transverse shear in the analysis. Studies pertaining to the flexural torsional buckling of composite laminated structures, taking in to account of the shear deformation effect have been carried out in the last two decades. Some of the recent studies are [9–12]. A recent study was also carried out by Sapountzakis and Dourakopoulos [13] using the boundary element method (BEM) in which the assumption of thin walled theory has been avoided. However, it can be noted that these studies are limited to Timoshenko beams with uniform and constant cross sections. To the best of the author's knowledge, publications on the solution to the flexural–torsional buckling analysis of generalized beams of arbitrarily shaped composite cross-section do not exist in open literature. In this investigation, the flexural torsional buckling has been carried out using an efficient hierarchical approach with variable kinematic 2D models successfully developed by Carrera [14,16] and Demasi and Carrera [15]. Carrera Unified Formulation (CUF) permits a systematic assessment of a large number of plate models, whose accuracy has been demonstrated to range from classic 2D models to quasi-3D descriptions for both the dynamic, stability and static stress analyses [17–21]. It may be noted that CUF is a hierarchical formulation which considers the order of the model as a free-parameter of the analysis, in other words, refined models are obtained with no need for *ad hoc* formulations. However, the 2D models used for composite structures are complex and computational expensive and thereby, a need for a much simpler model was brought out.

Because of their simplicity and computational efficiency over the two dimensional theories, the 1D higher order beam theories are also being used to analyze the structural behavior of slender bodies. Studies using the 1D finite element based hierarchical beam theory which incorporates the different expansion orders within the framework of Carrera Unified Formulation (CUF) have been carried out recently. This unified theory was first proposed by Carrera and Giunta [22]. Displacement-based theories that account for non-classical effects, such as transverse shear, in- and out-of-plane warping of the cross-section, can be formulated without the need of any assumption for warping functions. Classical models, such as Euler-Bernoulli and Timoshenko beam theories

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can be retrieved as particular cases. The authors found that the results of one-dimensional CUF models match very well with that obtained using the three-dimensional models. Carrera et al. [23] studied the systematic implementation of the finite elements in the CUF and found out that the finite element based CUF models were able to estimate accurate displacement/strain/stress distributions over the cross-section as well as over particular geometrical features such as corners and voids. Carrera and Petrolo [24] investigated the influence of higher order terms in the refined beam theories for static analyses and recommended the use of full CUF for practical purposes and for nonclassical geometries and loading conditions. Carrera et al. [25] investigated the free vibration analysis of beams with arbitrary geometries using the CUF. A brief review about developments in finite element formulations for vibration analysis of thin and thick laminated beams was also provided. The authors found out that the hierarchical beam models based on CUF were capable of detecting 3-D effects on the vibration modes as well as predicting shell-type vibration modes in case of thin walled beam sections.

This work presents the buckling analysis of thin walled structures by the 1D finite element higher-order models obtained within the framework of the CUF. In the present study, the refined beam theories are obtained on the basis of Taylor-type expansions. The finite element analysis has been chosen to easily handle arbitrary geometries as well as boundary conditions. Several beams are analyzed to illustrate the efficacy of the present formulation. In particular, buckling behavior of laminated composite beam and laminated flat panels are investigated and various types of buckling modes are observed.

2. Refined beam models and related FE formulations

2.1. Definitions

The adopted coordinate frame is presented in Fig. 1. The beam boundaries over y are $0 \leqslant y \leqslant L$. The displacement vector is:

$$\mathbf{u}(x,y,z) = \{ u_x \quad u_y \quad u_z \}^T \tag{1}$$

The superscript "T" represents the transposition operator. Stress, $\boldsymbol{\sigma}$, and strain, $\boldsymbol{\epsilon}$, components are grouped as follows:

$$\begin{aligned} \boldsymbol{\sigma}_p &= \{ \sigma_{zz} \quad \sigma_{xx} \quad \sigma_{zx} \}^T, \quad \boldsymbol{\epsilon}_p = \{ \epsilon_{zz} \quad \epsilon_{xx} \quad \epsilon_{zx} \}^T \\ \boldsymbol{\sigma}_n &= \{ \sigma_{zy} \quad \sigma_{xy} \quad \sigma_{yy} \}^T, \quad \boldsymbol{\epsilon}_n = \{ \epsilon_{zy} \quad \epsilon_{xy} \quad \epsilon_{yy} \}^T \end{aligned} \tag{2}$$

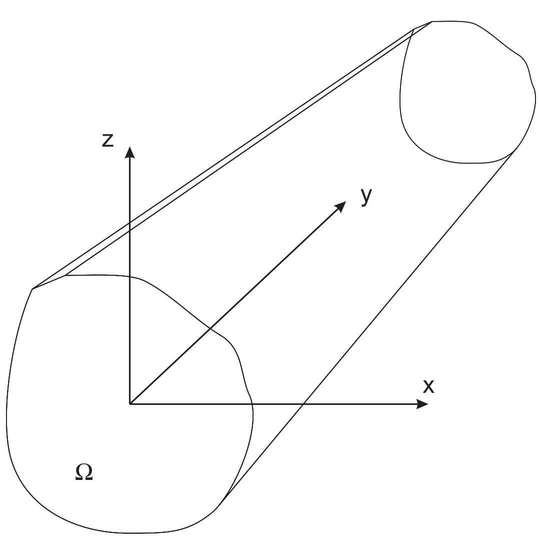

Fig. 1. Coordinate frame of the beam model.

The subscript "n" stands for terms laying on the cross-section, while "p" stands for terms laying on planes which are orthogonal to Ω. Linear strain–displacement relations are used:

$$\begin{aligned} \boldsymbol{\epsilon}_p &= \boldsymbol{D}_p \mathbf{u} \\ \boldsymbol{\epsilon}_n &= \boldsymbol{D}_n \mathbf{u} = (\boldsymbol{D}_{n\Omega} + \boldsymbol{D}_{ny})\mathbf{u} \end{aligned} \tag{3}$$

with:

$$\boldsymbol{D}_p = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix}, \quad \boldsymbol{D}_{n\Omega} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & 0 \end{bmatrix}, \quad \boldsymbol{D}_{ny} = \begin{bmatrix} 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & 0 & 0 \\ 0 & 0 & \frac{\partial}{\partial y} \end{bmatrix} \tag{4}$$

The Hooke law is exploited:

$$\boldsymbol{\sigma} = \widetilde{\boldsymbol{C}}\boldsymbol{\epsilon} \tag{5}$$

According to Eq. (2), the previous equation becomes:

$$\begin{aligned} \boldsymbol{\sigma}_p &= \widetilde{\boldsymbol{C}}_{pp}\boldsymbol{\epsilon}_p + \widetilde{\boldsymbol{C}}_{pn}\boldsymbol{\epsilon}_n \\ \boldsymbol{\sigma}_n &= \widetilde{\boldsymbol{C}}_{np}\boldsymbol{\epsilon}_p + \widetilde{\boldsymbol{C}}_{nn}\boldsymbol{\epsilon}_n \end{aligned} \tag{6}$$

The material matrices $\widetilde{\boldsymbol{C}}_{pp}, \widetilde{\boldsymbol{C}}_{nn}, \widetilde{\boldsymbol{C}}_{pn}$ and $\widetilde{\boldsymbol{C}}_{np}$ are:

$$\widetilde{\boldsymbol{C}}_{pp} = \begin{bmatrix} \widetilde{C}_{11} & \widetilde{C}_{12} & \widetilde{C}_{16} \\ \widetilde{C}_{12} & \widetilde{C}_{22} & \widetilde{C}_{26} \\ \widetilde{C}_{16} & \widetilde{C}_{26} & \widetilde{C}_{66} \end{bmatrix}, \quad \widetilde{\boldsymbol{C}}_{nn} = \begin{bmatrix} \widetilde{C}_{55} & \widetilde{C}_{45} & 0 \\ \widetilde{C}_{45} & \widetilde{C}_{44} & 0 \\ 0 & 0 & \widetilde{C}_{33} \end{bmatrix},$$

$$\widetilde{\boldsymbol{C}}_{pn} = \widetilde{\boldsymbol{C}}_{np}^T = \begin{bmatrix} 0 & 0 & \widetilde{C}_{13} \\ 0 & 0 & \widetilde{C}_{23} \\ 0 & 0 & \widetilde{C}_{36} \end{bmatrix} \tag{7}$$

For the sake of brevity, the dependence of coefficients $[\widetilde{C}]_{ij}$ versus Young's modulus (E) and Poisson's ratio (ν) is not reported here. It can be found in standard texts [26].

In the framework of the CUF, the displacement field is assumed as an expansion in terms of generic functions, F_τ:

$$\mathbf{u} = F_\tau \mathbf{u}_\tau, \qquad \tau = 1, 2, \ldots, M \tag{8}$$

where F_τ are functions of coordinates x and z on the cross-section. $\mathbf{u}_\tau$ is the displacement vector and M stands for the number of terms of the expansion. According to the Einstein notation, the repeated subscript τ indicates summation. In the next subsections, advanced theories based on Taylor series expansion for cross sectional displacement coupled with finite elements representing the translational displacement field and the theory based on Lagrange polynomials are presented.

2.2. Taylor 1D CUF

Using the Maclaurin expansion that uses as basis the 2D polynomials $x^i\, z^j$, where i and j are positive integers. Considering the expansion up to the quadratic terms, Eq. (8) can be written as:

Table 1
First three dimensionless bending buckling loads ($P = P_o \frac{12}{\pi^2}\left(\frac{L}{a}\right)^2$, where P_o is the actual buckling load) for different refined beam theories using 10 B4 elements for an isotropic beam (L/a = 20).

Critical load (P)	P_1	P_2	P_3
Ref. [32]	0.9919	3.873	8.387
EBBT	0.995	3.956	8.813
TBT	0.9900	3.875	8.422
N = 1	0.9925	3.884	8.437
N = 2	0.9927	3.885	8.444
N = 3	0.9918	3.873	8.387