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Buckling of conical composite shells

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ABSTRACT

A semi-analytical approach is proposed to obtain the linear buckling response of conical composite shells under axial compression load. A first order shear deformation shell theory along with linear strain–displacement relations is assumed. Using the principle of minimum total potential energy, the governing equilibrium equations are found and Ritz method is applied to solve them. Parametric study is performed by finding the effect of cone angle and fiber orientation on the critical buckling load of the conical composite shells.

1. Introduction

Conical composite shells have wide applications in aerospace/aeronautical, naval and civil structures. This extensive application of conical composite shells in industry calls for efficient tools to analyze the mechanical behavior of these structures. In many applications the primarily concern is the stability of the structure and analytical solution is necessary to predict the critical buckling load.

There have been extensive studies on the buckling of isotropic conical shells under axial load and external pressure [1–8]. Seide [1] proposed a formula for buckling of isotropic conical shell which is independent of boundary conditions and best fits the behavior of long shells. Baruch et al. [8] investigated the stability of simply supported isotropic conical shells under axial load for four different sets of in-plane boundary conditions using Donnell-type theory.

However, although laminated composite materials have found extensive industrial applications during the last decades, only few studies have been published targeting the buckling behavior of conical composite shells. Using Donnell-type shell theory, Tong and Wang [9] proposed a power series based solution for buckling analysis of laminated conical shells under axial compressive load and external pressure. Li [10] considered the stability of composite stiffened shell under axial compression load. He assumed classical lamination theory and used Rayleigh–Ritz approximation to solve the governing equations. Sofiyev [11] studied the buckling of orthotropic composite conical shell incorporating the effect of thickness variation and time dependent external pressure. Donnell-type shell theory was assumed in his work and Galerkin method and variational technique were applied to obtain the solution. Static, free vibration and buckling analysis of laminated conical shell using finite element method based on higher order shear deformation theory was carried out by Pinto Correia et al. [12]. The effect of variations of the stiffness coefficients on the buckling of laminated conical shells was studied by Goldfeld and Arbocs [13] using classical shell theory and computer code STAGS-A. Patel et al. [14] studied postbuckling characteristics of angle-ply laminated conical shells subjected to torsion, external pressure, axial compression, and thermal loading using the finite element approach.

In this paper, axisymmetric and non-axisymmetric formulations for buckling analysis of conical composite shells subjected to axial compression load are developed and semi-analytical solution using Ritz method is obtained. The effect of transverse shear deformation is taken into account since the classical theory of shells was shown not to be accurate enough for moderately thick laminated shells and where the material anisotropy is severe [12,15]. Parametric study is carried out to reveal the influence of the semi-cone angle and the fiber orientation on critical buckling load.

2. Formulation

The principle of minimum total potential energy in conjunction with variational techniques is employed to derive the governing equations for general laminated thin-walled conical shells. Linear strain–displacement relation is assumed. Furthermore, using first-order shear deformation shell theory, Kirchhoff hypothesis denoting that the transverse normal to the mid-surface remains perpendicular to it after deformation, is relaxed.

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2.1. Kinematics

The coordinate system associated with conical shell is shown in Fig. 1. Based on the shear deformation shell theory assumption, the displacement field can be written as [16].

$$\begin{aligned} U(x,\theta,z) &= u(x,\theta) + z\beta_x(x,\theta) \\ V(x,\theta,z) &= v(x,\theta) + z\beta_\theta(x,\theta) \\ W(x,\theta,z) &= w(x,\theta) \end{aligned} \tag{1}$$

In these equations u, v and w are the displacements of mid-surface and β_x and β_θ are the rotations of a normal to the mid-surface about θ and x axis respectively.

The strain–displacement relations can be stated in terms of mid-surface strain and the curvature of the shell as

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z\varepsilon'_x \\ \varepsilon_\theta &= \varepsilon_\theta^0 + z\varepsilon'_\theta \\ \gamma_{x\theta} &= \gamma_{x\theta}^0 + z\gamma'_{x\theta} \\ \gamma_{xz} &= \gamma_{xz}^0 \\ \gamma_{\theta z} &= \gamma_{\theta z}^0 \end{aligned} \tag{2}$$

where

$$\begin{aligned} &\varepsilon_x^0 = \frac{\partial u}{\partial x} \quad \varepsilon_\theta^0 = \frac{1}{x\sin(\alpha)}\left(\frac{\partial v}{\partial \theta} + u\sin(\alpha) + w\cos(\alpha)\right) \\ &\varepsilon'_x = \frac{\partial \beta_x}{\partial x} \quad \varepsilon'_\theta = \frac{1}{x\sin(\alpha)}\left(\frac{\partial \beta_\theta}{\partial \theta} + \theta_x \sin(\alpha)\right) \\ &\gamma_{x\theta}^0 = \frac{\partial v}{\partial x} - \frac{1}{x\sin(\alpha)}\left(v\sin(\alpha) - \frac{\partial u}{\partial \theta}\right) \\ &\gamma_{xz}^0 = \frac{\partial w}{\partial x} + \beta_x \quad \gamma'_{x\theta} = \frac{\partial \beta_\theta}{\partial x} + \frac{1}{x\sin(\alpha)}\frac{\partial \beta_x}{\partial \theta} - \frac{\beta_\theta}{x} \\ &\gamma_{\theta z}^0 = \frac{1}{x\sin(\alpha)}\frac{\partial w}{\partial \theta} - \frac{v}{x\sin(\alpha)}\cos(\alpha) + \beta_\theta \end{aligned} \tag{3}$$

α in above equations is semi-cone angle.

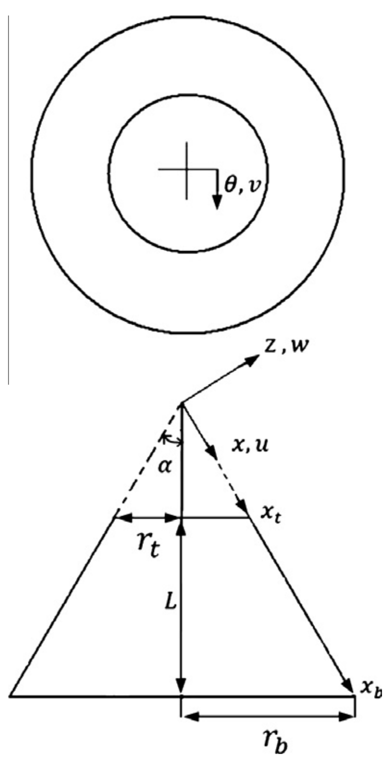

Fig. 1. Conical shell coordinate system.

2.2. Constitutive law

Assuming orthotropic properties for each layer and neglecting the transverse normal stress, force and moment resultants are related to strains by laminated stiffness coefficients as follows [17]:

$$\begin{aligned} &\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_\theta^0 \\ \gamma_{x\theta}^0 \\ \varepsilon'_x \\ \varepsilon'_\theta \\ \gamma'_{x\theta} \end{Bmatrix} \\ &\begin{Bmatrix} Q_\theta \\ Q_x \end{Bmatrix} = K_s \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{\theta z}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \end{aligned} \tag{4}$$

2.3. Total potential energy

The total potential energy of the conical shell subjected to an axial compressive in-plane load consists of the strain energy and the potential energy due to uniaxial compressive force per unit length. It can be written as

$$\Pi = U + W \tag{5}$$

in which U represents the strain energy, while W is the work of the applied in-plane compressive force and are defined as

$$\begin{aligned} &U = \frac{1}{2}\iint_{x\theta} (N_x\varepsilon_x^0 + N_\theta\varepsilon_\theta^0 + N_{x\theta}\gamma_{x\theta}^0 + M_x\varepsilon'_x + M_\theta\varepsilon'_\theta + M_{x\theta}\gamma'_{x\theta} \\ &+ Q_y\gamma_{xz}^0 + Q_x\gamma_{\theta z}^0) r d\theta dx \quad W = \frac{1}{2}\iint_{x\theta} \widehat{N}_{xx}\left(\frac{\partial w}{\partial x}\right)^2 r d\theta dx \end{aligned} \tag{6}$$

The principle of minimum total potential energy which can give the governing equations of the conical shell subjected to compressive axial load can be stated as

$$\delta\Pi = \delta U + \delta W = 0 \tag{7}$$

where δU and δW are first variation of strain energy and work of the applied in-plane compressive force, respectively.

2.4. Ritz method

Ritz method is a powerful tool to determine approximate solutions of the governing equations by using the principle of minimum total potential energy (Eq. (7)). This method has an advantage of giving directly the solution from variational statement by bypassing the equilibrium equations [18].

The approximate solutions for displacements are assumed in series form as

$$\begin{aligned} &[u(x,\theta), v(x,\theta), w(x,\theta), \beta_x(x,\theta), \beta_\theta(x,\theta)] \\ &\quad = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\Phi_{ij}^u, \Phi_{ij}^v, \Phi_{ij}^w, \Phi_{ij}^{\beta_x}, \Phi_{ij}^{\beta_\theta}\right] \end{aligned} \tag{8}$$

where Φ_{ij}^u, Φ_{ij}^v, Φ_{ij}^w, $\Phi_{ij}^{\beta_x}$ and $\Phi_{ij}^{\beta_\theta}$ are approximation functions and they need to satisfy convergence and completeness requirements [18].

Considering Eq. (8) and using Eqs. (2)–(6) and rewriting them in a matrix form, one can derive the first variation of the strain energy and work of the applied force by performing repeated integration by part as

$$\begin{aligned} \delta U &= \iint_{x\theta} \delta[C]^T[\Phi]^T[B]^T[F][B][\Phi][C] r d\theta dx \\ \delta W_s &= \iint_{x\theta} \delta[C]^T[\Phi]^T[\hat{B}][\Phi][C] r d\theta dx \end{aligned} \tag{9}$$