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Local buckling behavior of FRP thin-walled beams: A mechanical model

Luigi Ascione, Valentino Paolo Berardi *, Antonella Giordano, Saverio Spadea

Department of Civil Engineering, University of Salerno, Italy

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ABSTRACT

A mechanical model able to predict the local buckling of pultruded FRP thin-walled beams and columns, taking into account the shear deformability of composite materials, is presented in this paper. The model is based on the individual analysis of the buckling of the components of the FRP profile, assumed as elastically restrained transversely isotropic plates. The analysis is developed within the hypotheses of small strains and moderate rotations.

1. Introduction

The use of pultruded Fiber Reinforced Polymer (FRP) thin-walled beams represents an interesting challenge in the field of Civil Engineering, in view of the well-known advantages that composite materials exhibit in comparison with more conventional ones.

Within this context, the design process at ultimate limit state is generally governed by buckling rather than strength limitations, due to both the thin-walled sectional geometry of the members and the relatively low shear stiffness of the composite materials [1–14]. Several studies on global buckling have demonstrated that shear deformation can affect the ultimate failure of thin-walled FRP members [15–18].

In addition, the analysis of local buckling failure in composite profiles still needs further investigation.

The experimental studies on this topic have highlighted a reasonable influence of local buckling on the failure load [19–21].

From a theoretical point of view, a design procedure based on a FEM analysis by using plate elements has been proposed in literature [20,21]. This method typically assumes a rigid behavior of junctions (i.e. flange–web junctions in the case of an open section), although experimental investigations have highlighted, on the contrary, a semi-rigid constitutive behavior [22].

Alternatively, an individual modeling of the profile plate components (i.e. flanges and web in the case of an open section) under the assumption of flexible junctions has been carried out by different authors [23–30] and adopted by Italian Guidelines [31]. Despite composite materials typically exhibiting low shear elasticity moduli, these studies are based on the approximated assumption of negligible shear deformability.

It follows that both approaches may lead to an overestimation of the beam stiffness and, consequently, of the buckling load.

In addition, it should be noted that the scientific contributions so far available in literature typically deal with the axial load case (columns), while the flexural load case needs to be analyzed in further detail. The present state of art has motivated the interest to contribute towards a better understanding of the problem.

The approach here proposed allows to predict the local buckling of pultruded FRP thin-walled beams and columns at ultimate limit state, taking into account the shear deformability of composite materials. The purpose is to provide an upgrade of the abovementioned individual buckling modeling, accounting for the shear deformability of the composite, while still considering each single plate component of the FRP profile to be elastically restrained along the junctions.

With this aim, the Mindlin–Reissner theory is developed in the field of small strains and moderate rotations, with it also being extended to elastic transversely isotropic materials.

The numerical analysis presented, performed by means of a weak formulation of the buckling problem within the finite element method, examines the case of an "I" simply supported beam subject to either an axial or transverse load.

The procedure is validated through a comparison with values numerically obtained by adopting the approach proposed in [19,20] for a beam under a constant axial load.

2. Kinematics

Let consider an "I" beam with the constant cross-section of Fig. 1. The adopted Cartesian reference frame has the origin in the centroid, G, of one of the bases of the beam, with the X and Y axes being coincident with the central axes of inertia of the

* Corresponding author. Tel.: +39 089 964084; fax: +39 089 968744.
E-mail addresses: l.ascione@unisa.it (L. Ascione), berardi@unisa.it (V.P. Berardi), angiordano@unisa.it (A. Giordano), sspadea@unisa.it (S. Spadea).

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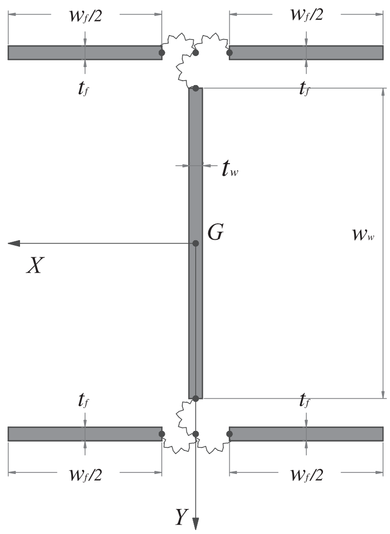

Fig. 1. Schematic representation of the pultruded profile cross section.

cross-section and the Z axis coincident with the longitudinal beam axis.

The cross section can be divided into five plate components, internally connected through flexible flange–web junctions (Fig. 1).

Thus, the single plate can be modeled as an elastically restrained transversely isotropic one.

Let denote the undeformed mid-plane of the plate with the symbol Ω_0. The total domain of the plate is $\Omega_0 \times (-h/2, +h/2)$. The boundary of the total domain consists of surfaces $S^+(Z = +h/2)$, $S^-(Z = -h/2)$ and $\Gamma = \partial\Omega_0 \times (-h/2, +h/2)$ (Fig. 2). Γ is a curved surface, with outwards normal $\hat{\mathbf{n}} = n_x\hat{\mathbf{e}}_{\mathbf{x}} + n_y\hat{\mathbf{e}}_y$, where n_x and n_y are the direction cosines of the unit normal.

The classical theory available for shear deformable plates, according to Mindlin and Reissner [32,33], is based on the following assumptions:

- straight lines perpendicular to the mid-plane (i.e. transverse normals) before deformation remain straight after deformation;
- the transverse normals do not experience elongation.

Therefore, the displacement field of the plate, referred to the coordinate system $\{O,x,y,z\}$ (Fig. 2), can be written in the following form:

$$u = u(x,y,z) = u_0(x,y) + \varphi_y(x,y) \cdot z, \tag{1a}$$

$$v = v(x,y,z) = v_0(x,y) - \varphi_x(x,y) \cdot z, \tag{1b}$$

$$w = w(x,y) = w_0(x,y), \tag{1c}$$

where (u_0, v_0, w_0) denote the displacements of a material point at the mid-plane of the plate and (φ_x, φ_y) are the rotations of transverse normals about the x-axis and y-axis, respectively.

In order to take into account geometric nonlinearity, the Green–Saint Venant strain tensor $\mathbf{E}$ is taken into consideration. In terms of displacement gradient, $\nabla\mathbf{u}$, $\mathbf{E}$ is:

$$\mathbf{E} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}} + \nabla\mathbf{u}^{\mathrm{T}}\nabla\mathbf{u}). \tag{2}$$

The displacement gradient is decomposed in its symmetric and skew parts, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\omega}$:

$$\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}}), \tag{3a}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \frac{\partial u_0}{\partial x} + z\frac{\partial \varphi_y}{\partial x} & \frac{1}{2}\left[\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z\left(\frac{\partial \varphi_y}{\partial y} - \frac{\partial \varphi_x}{\partial x}\right)\right] & \frac{1}{2}\left(\varphi_y + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left[\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + z\left(\frac{\partial \varphi_y}{\partial y} - \frac{\partial \varphi_x}{\partial x}\right)\right] & \frac{\partial v_0}{\partial y} - z\frac{\partial \varphi_x}{\partial y} & \frac{1}{2}\left(-\varphi_x + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\varphi_y + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(-\varphi_x + \frac{\partial w}{\partial y}\right) & 0 \end{bmatrix}, \tag{3b}$$

$$\boldsymbol{\omega} = \frac{1}{2}(\nabla\mathbf{u} - \nabla\mathbf{u}^{\mathrm{T}}), \tag{4a}$$

$$\boldsymbol{\omega} = \begin{bmatrix} 0 & \frac{1}{2}\left[\frac{\partial u_0}{\partial y} - \frac{\partial v_0}{\partial x} + z\left(\frac{\partial \varphi_y}{\partial y} + \frac{\partial \varphi_x}{\partial x}\right)\right] & \frac{1}{2}\left(\varphi_y - \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left[-\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - z\left(\frac{\partial \varphi_y}{\partial y} + \frac{\partial \varphi_x}{\partial x}\right)\right] & 0 & -\frac{1}{2}\left(\varphi_x + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(-\varphi_y + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\varphi_x + \frac{\partial w}{\partial y}\right) & 0 \end{bmatrix}, \tag{4b}$$

Tensors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\omega}$ are the infinitesimal strain tensor and the infinitesimal rotation tensor, respectively.

Thus, the Green–Saint Venant strain tensor $\mathbf{E}$ can be also written in terms of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\omega}$:

$$\mathbf{E} = \boldsymbol{\varepsilon} + \frac{1}{2}(\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\omega} + \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\varepsilon} + \boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega}). \tag{5}$$

According to the Mindlin–Reissner theory, the following hypothesis is assumed:

$$\alpha = \|\nabla\mathbf{u}\| \ll 1 \text{ in } \Omega_0 \times (-h/2, +h/2). \tag{6}$$

Within such a theory, the infinitesimal strain tensor components are assumed to be of the order α:

$$\boldsymbol{\varepsilon} = \boldsymbol{O}(\alpha). \tag{7}$$

On the contrary, the order of magnitude of the infinitesimal rotation tensor components are assumed to be of the order $\alpha^{1/2}$:

$$\boldsymbol{\omega} = \boldsymbol{O}(\alpha^{1/2}), \tag{8}$$

thus the local rotation is defined as "moderately large" [34].

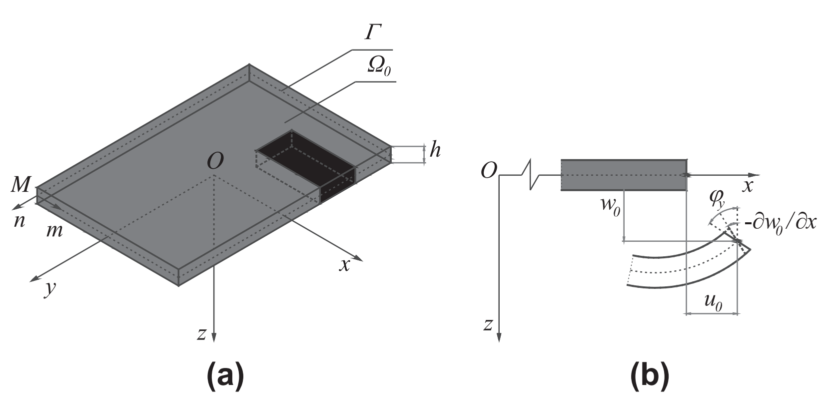

Fig. 2. (a) Global $\{O,x,y,z\}$ and local $\{M,m,n,z\}$ coordinate system of the plate and (b) undeformed and deformed geometry of the shear deformable plate.