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Buckling analysis of functionally graded carbon nanotube-reinforced composite plates using the element-free *kp*-Ritz method

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ABSTRACT

This paper presents the buckling analysis of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) plates under various in-plane mechanical loads, using the element-free *kp*-Ritz method. The first-order shear deformation plate theory is applied and a set of mesh-free kernel particle functions are used to approximate two-dimensional displacement fields. Effective properties of materials of the plates reinforced by single-walled carbon nanotubes (SWCNTs) are estimated through a micromechanical model based on either the Eshelby–Mori–Tanaka approach or the extended rule of mixture. Comparison study and numerical simulations with various parameters are conducted to assess efficacy and accuracy of the present method for analysis of buckling of SWCNT-reinforced composite plates. Results demonstrate that the change of carbon nanotube volume fraction, plate width-to-thickness ratio, plate aspect ratio, loading condition and temperature have pronounced effects on buckling strength of CNTRC plates as well as the boundary condition.

1. Introduction

Recently, carbon nanotubes (CNTs), as a new type of advanced materials, have attracted a great deal of interest of researchers. Due to their extremely attractive mechanical, electrical and thermal properties, CNTs have promising applications in polymer composites as a potential reinforcement and multi-functional element [1,2]. Therefore, the introduction of CNTs into a polymer matrix may greatly improve mechanical properties of the resulting nanocomposites, such as tensile strength and elastic modulus [3].

Most investigations of carbon nanotubes-reinforced composites (CNTRCs) have focused on material properties and researchers have discovered that mechanical, electrical and thermal properties of polymer composites can be considerably improved by adding small amounts of CNTs. Odegard et al. [4] presented a constitutive modeling of nanotubes-reinforced polymer composites with nanotubes/polymer interface modeled as an effective continuum fiber by using an equivalent-continuum model. Gary et al. [5] obtained the effective elastic properties of CNTRCs through a variety of micromechanics techniques with the effective properties of CNTs calculated utilizing a composite cylinders micromechanics technique as a first step in a two-step process. Fidelus et al. [6] examined the thermo-mechanical properties of epoxy-based nanocomposites based on low weight fractions of randomly oriented single- and multi-walled carbon nanotubes. Han and Elliot [7] presented classical molecular dynamics (MD) simulations of model polymer/CNT composites constructed by embedding a single wall (10, 10) CNT into two different amorphous polymer matrices. By using MD method, the stress–strain behavior of carbon nanotube-reinforced Epon862 composites was also studied by Zhu et al. [8].

Although these studies are quite useful, the ultimate purpose of development of this advanced material is to explore potential applications of CNTRCs in actual structures, such as CNT-reinforced beams, plates or shells. Wuite and Adali [9] studied deflection and stress behaviors of nanocomposite reinforced beams using a multiscale analysis. Their results showed that reinforcement by adding a small proportion of nanotube leads to significant improvement in beam stiffness. Vodenitcharova and Zhang [10] presented analyses of pure bending and bending-induced local buckling of a nanocomposite beam based on a continuum mechanical model and found that single-walled carbon nanotube (SWCNT) buckles at smaller bending angles and greater flattening ratios in thicker matrix layers. Formica et al. [11] investigated vibration behaviors of CNTRC plates by employing an equivalent continuum model based on the Eshelby–Mori–Tanaka approach. Based on the classical laminated plate theory and third-order shear deformation theory, Arani et al. [12] analytically and numerically studied buckling behaviors of laminated composite plates. The optimal orientation of CNTs to achieve the highest critical load and corresponding mode shape

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were calculated for different kinds of boundary conditions as well as aspect ratios of the plates. Shen [13,14] presented a postbuckling analysis of cylindrical shells reinforced by SWCNTs subjected to axial compression and lateral or hydrostatic pressure in thermal environments. Results revealed that the mid-plane symmetric functionally graded distribution of reinforcements can increase the buckling load as well as postbuckling strength of the shells. They also confirmed that the postbuckling equilibrium path for both FG- and UD-CNTRC cylindrical shells under axial compression is unstable.

In traditional nanocomposites, the resulting mechanical, thermal, or physical properties do not vary spatially at the macroscopic level since nanotubes distribute either uniformly or randomly in the composites. Stimulated by the concept of functionally graded materials (FGMs) with properties that vary spatially according to a certain non-uniform distribution of the reinforcement phase, Shen [15] presented a non-linear bending analysis of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) plates in thermal environments using a two-step perturbation technique. The results showed that non-linear bending behaviors of CNTRC plate can be considerably improved as CNTs distribute functionally in the matrix. By using the finite element method (FEM), analyses of bending and free vibration were carried out for various types of functionally graded CNTRC plates by Zhu et al. [16]. They discovered that CNT reinforcements distributed close to top and bottom are more efficient than those distributed near the mid-plane for increasing the stiffness of CNTRC plates. Based on the Timoshenko beam theory, non-linear free vibrations of functionally graded CNTRC beams were analyzed with the Ritz method and direct iterative technique by Ke et al. [17]. They found linear and non-linear frequencies of FG-CNTRC beam with symmetrical distribution of CNTs higher than those of beams with uniform or asymmetrical distribution of CNTs. For improving buckling and postbuckling behaviors of CNTRC structures, Shen and Zhang [18] investigated thermal buckling and postbuckling behaviors of FG-CNTRC plates and found that CNTRC plate with intermediate nanotube volume fraction does not have intermediate buckling temperature and initial thermal postbuckling strength.

In the present work, a buckling analysis of FG-CNTRC plates under different in-plane loading conditions in thermal environment is presented using the element-free *kp*-Ritz method, which has already been successfully applied in many fields [19–22]. Two kinds of CNTRC plates, namely, uniformly distributed (UD) and functionally graded (FG) CNTRC plates, are considered. The eigenvalue equations of buckling analysis of CNTRC plates are obtained by applying the Ritz procedure to the energy function of the system. In order to improve computational efficiency and avoid shear locking for very thin plates, a stabilized conforming nodal integration approach is used to calculate the plate bending stiffness, and the shear and membrane terms are evaluated using a direct nodal integration method. Effects of CNT volume fraction, plate width-to-thickness ratio, plate aspect ratio, boundary condition, in-plane loading condition and temperature change on buckling strength of CNTRC plates are examined in detail.

2. Carbon nanotube-reinforced composites

Three types of distributions of CNTs in CNTRC plates with length a, width b and thickness h are considered (Fig. 1). UD denotes the uniform distribution and the other two types of functionally graded distributions of CNTs are represented by FG-O and FG-X. The plates are assumed to be made of a mixture of SWCNTs and the matrix. The matrix is assumed to be isotropic and material properties of SWCNTs are chirality-, size- and temperature-dependent [23–26]. Distributions of CNTs along the thickness direction of UD- and the other two types FG- of CNTRC plates are assumed to be as follows:

$$V_{CNT}(z) = \begin{cases} V^*_{CNT} & (\text{UD} \quad \text{CNTRC}), \\ 2\left(1 - \frac{2|z|}{h}\right)V^*_{CNT} & (\text{FG-O} \quad \text{CNTRC}), \\ 2\left(\frac{2|z|}{h}\right)V^*_{CNT} & (\text{FG-X} \quad \text{CNTRC}), \end{cases} \tag{1}$$

where

$$V^*_{CNT} = \frac{w_{CNT}}{w_{CNT} + (\rho^{CNT}/\rho^m) - (\rho^{CNT}/\rho^m)w_{CNT}}, \tag{2}$$

where w_{CNT} is the fraction of mass of CNTs and ρ^m and ρ^{CNT} are densities of the matrix and CNTs, respectively. V_{CNT} and V^*_{CNT} are the CNT volume fractions of FG- and UD-CNTRCs and we assume $V_{CNT} = V^*_{CNT}$, that means UD-CNTRC plate and the other two types of FG-CNTRC plates have the same mass volume of CNTs.

As the load transfer between the nanotube and matrix is less than perfect, several micromechanical models have been

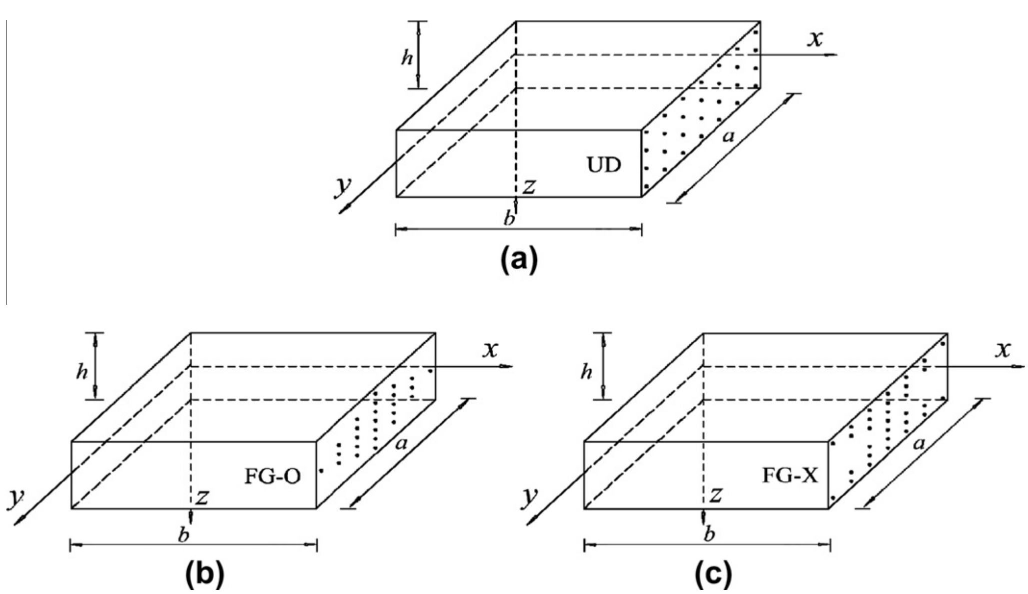

Fig. 1. Configurations of carbon nanotube reinforced composite plates. (a) UD CNTRC plate; (b) FG-O CNTRC plate; (c) FG-X CNTRC plate.