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Masonry arches strengthened with composite unbonded tendons

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ABSTRACT

In this paper an analytical model to evaluate the structural behavior of masonry arches and vaults strengthened with composite unbonded tendons placed at the extrados is presented. The tendons are fixed at the imposts. The model is formulated under the assumption of finite displacements. The displaced equilibrium configurations are identified by the stationarity of the potential of the acting forces. It is shown that when the tendon is not pretensioned an increase of the arch collapse load can be achieved only if the axial stiffness of the tendon is sufficiently large. Instead if the tendon is pretensioned an increase of the load that induces the first displacement of the arch is always achieved. If the stiffness of the tendon is sufficiently large the collapse load will be greater than the load that produces the first displacement of the arch.

1. Introduction

The load carrying capacity of masonry arches and vaults (Fig. 1) can be increased following two different strengthening strategies.

The first one consists in modifying the collapse mechanism of the structure using strengthening materials externally bonded at the intrados or the extrados surfaces. The strengthening material at the intrados (extrados) surface avoids the formation of hinges at the extrados (intrados) surface [1–3]. The modification of the collapse mechanism, that is related to the modification of the hinges positions, produces a consequent increase of the collapse load (Fig. 1b). Externally bonded fiber reinforced polymers (FRP) and fiber reinforced cementitious matrix (FRCM) are commonly used as strengthening materials within this context. In this case the key issue is the bond between the strengthening material and the support. The definition of the bond-slip relation is essential in the structural modeling of historical masonry constructions strengthened with composite materials, particularly when their seismic capacity is evaluated through nonlinear analyses [4,5]. This topic has been extensively investigated in the case of the FRP materials in many papers (e.g. [6,7]) while in the case of FRCM strengthening materials only a few research works have been performed [8–14].

The second strengthening strategy consists in the increase of the collapse load of the structure obtained contrasting the evolution of the collapse mechanism with an unbonded tendon placed at the extrados. In this case the evolution of the mechanism causes a tendon elongation and consequently an increase of the tendon tensile force which in turn contrasts the mechanism itself (Fig. 1c).

Experimental investigations relative to this strengthening strategy are presented in [15,16]. In these studies steel tendons were adopted. A strengthening of this type can be alternatively attained using tendons made of composite materials that can slip without friction inside a sheath. Rebars and tendon made of composite materials, although widely used in the cases of reinforcing and prestressing of concrete structures [17–19], have not yet been tested for the case analyzed in the present study.

An analytical model relative to a semicircular arch made of no tension material strengthened with an unbonded elastic tendon placed at the extrados is presented. The tendon is fixed at the arch imposts and it is supposed to slip without friction upon the extrados surface.

2. Analytical model

The mechanical model is formulated according to the following assumptions:

- the arch is made of no tension material [20–23], having an infinite compressive strength and stiffness; consequently a collapse mechanism is achieved both in the unstrengthened and strengthened configurations; this assumption results consistent with the actual structural behavior of unstrengthened masonry arches [24];
- the imposts are fixed for the presence of chains or massive abutments;

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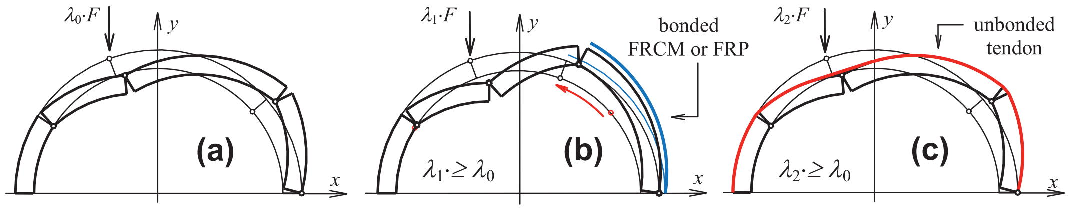

Fig. 1. Collapse of an (a) unstrengthened, (b) strengthened with externally bonded material and (c) strengthened with an unbonded tendon masonry arch.

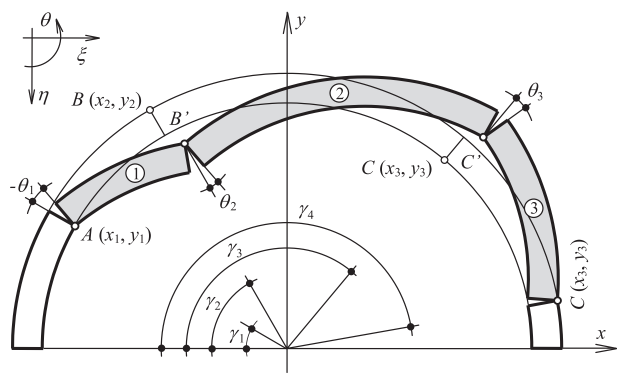

Fig. 2. Initial and generic configurations of the structure.

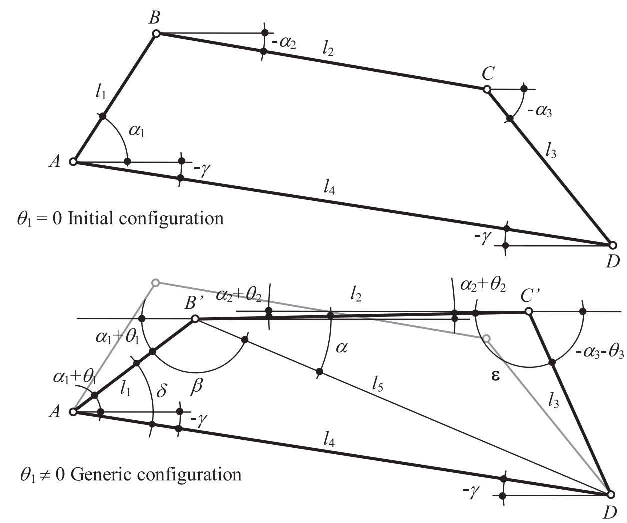

Fig. 3. Definition of geometric parameters.

- the tendon slips without friction at the extrados of the arch and it is fixed at the imposts;
- the tendon is made of a linear elastic composite material;
- the load configuration is constituted by the dead load (self weight and infill) and by an increasing vertical force;
- the unstrengthened arch does not collapse under the dead loads only;
- the collapse mechanism of the unstrengthened arch is characterized by four hinges placed as represented in Fig. 2;
- the hinges position minimizes the load producing the first displacement of the structure and does not change during its further displacements; this assumption is consistent with the fact that in a masonry arch the hinges can take place only at the mortar joints; this assumption could be removed in future studies on this topic.

The presented analytical model can be easily extended to a generic load configuration in which all the live loads are proportional to a load multiplier.

2.1. Geometry

A semicircular masonry arch (Fig. 2) with extrados and intrados radii R_e and R_i, respectively, a span $L_a = R_e + R_i$ and a thickness $s = R_e - R_i$ is considered. Four hinges having coordinates (x_j, y_j), $j = 1, \ldots, 4$ in the system represented in Fig. 2 are considered.

The hinges positions are identified by the angles γ_j, $j = 1, \ldots, 4$. The hinges position is such that if the hinge j is placed at the extrados, the hinges $j-1$ and $j+1$ are placed at the intrados and the quadrilateral having vertexes at the hinges is convex (Fig. 3).

Due to the presence of the hinges, the arch is divided into five rigid blocks. The three rigid blocks that can rotate are numbered so that the jth rigid block is delimited by the cross sections where the jth and $(j+1)$th hinges are located. In this configuration the arch is a system with one degree of freedom. The rotation of the first rigid block θ_1 is assumed as degree of freedom that identifies the generic configuration of the system. The rotation of the second and the third rigid blocks are θ_2 and θ_3. Counterclockwise rotations are considered positive. With reference to Figs. 2 and 3, the functions $\theta_2 = \theta_2(\theta_1)$ and $\theta_3 = \theta_3(\theta_1)$ are first determined, than the displacements of some points of interest are determined as a function of these rotations. l_1, l_2, l_3, l_4 are the sides of the quadrilateral having the vertex at the hinges. The lengths l_1, l_2, l_3, l_4 and the angles α_1, α_2, α_3, γ (Fig. 3) have to be preliminarily determined. If in the displaced configuration identified by $\theta_1 \neq 0$ the quadrilateral with the vertex at the hinges is still convex, it results

$$\theta_2(\theta_1) = \alpha_1 + \theta_1 + \alpha(\theta_1) + \beta(\theta_1) - \alpha_2 - \pi \tag{1}$$

where the angles $\alpha(\theta_1)$ and $\beta(\theta_1)$ are obtained as follows:

$$\alpha(\theta_1) = a\cos\frac{l_2^2 + l_5^2 - l_3^2}{2l_2l_5} \quad \beta(\theta_1) = a\cos\frac{l_1^2 + l_5^2 - l_4^2}{2l_1l_5} \tag{2}$$

where the length l_5 (Fig. 3) is

$$l_5(\theta_1) = \sqrt{l_1^2{+}{}_4^2 - 2l_1l_4\cos\delta} = \sqrt{l_1^2{+}{}_4^2 - 2l_1l_4\cos(\alpha_1 + \theta_1 - \gamma)} \tag{3}$$

The rotation of the second rigid block $\theta_2 = \theta_2(\theta_1)$ can be then expressed as

$$\theta_2(\theta_1) = \alpha_1 + a\cos\frac{l_1^2 + l_2^2 - l_3^2 + l_4^2 - 2l_1l_4a\cos(\alpha_1 + \theta_1 - \gamma)}{2l_2\sqrt{l_1^2 + l_4^2 - 2l_1l_4\cos(\alpha_1 + \theta_1 - \gamma)}} + a\cos\frac{2l_1^2 - 2l_1l_4\cos(\alpha_1 + \theta_1 - \gamma)}{2l_1\sqrt{l_1^2 + l_4^2 - 2l_1l_4\cos(\alpha_1 + \theta_1 - \gamma)}} - \alpha_2 - \pi \tag{4}$$

As concerns $\theta_3 = \theta_3(\theta_1)$, it results

$$\theta_3(\theta_1) = \varepsilon(\theta_1) + \alpha_2 + \theta_2(\theta_1) - \alpha_3 - \pi \tag{5}$$

where