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Modelling of attenuation of Lamb waves using Rayleigh damping: Numerical and experimental studies

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ABSTRACT

In this study, the Lamb mode attenuation constants were derived in terms of the attenuation coefficient, group velocity and central frequency of excitation of the Lamb mode, using the Rayleigh damping model. Attenuation of Lamb waves, both fundamental symmetric and anti-symmetric modes, propagating through viscoelastic media (cross-ply glass/epoxy laminate) was modelled using the Finite Element Method. Numerically simulated attenuation of Lamb waves using Lamb mode attenuation constants was found to be in good agreement with the assumed attenuation. Experiments were performed on a quasi-isotropic laminate, employing air-coupled ultrasonic transducers, to measure the attenuation coefficient. Lamb mode attenuation constants, computed using the attenuation coefficient, were used to model the attenuation of the Lamb mode in quasi-isotropic laminates. Numerically simulated amplitude variation was found to be in good agreement with that computed from experiments.

1. Introduction

Lamb waves are ultrasonic waves which propagate through thin plate-like structures. These waves are employed for Non-Destructive Evaluation (NDE) and Structural Health Monitoring (SHM) applications [14]. Features of Lamb waves used for NDE and SHM are Time-of-Flight (ToF), mode conversion/generation, change in amplitude/attenuation, velocity, etc. The attenuation of Lamb waves is essentially due to the material's viscoelastic properties. This attenuation is a critical parameter in the selection of a particular mode for long range NDE of viscoelastic materials like laminated composites. Attenuation is often neglected in wave propagation analysis because of complexities involved in modelling.

Kazys et al. [5] investigated the interaction of guided waves with welds, defects and other deformities in steel plates loaded by a liquid. In this work, the attenuation of guided wave was calculated from the spectra ratio of the signals captured by receivers placed at two different locations. It was observed that the frequency dependent attenuation of the signal transmitted through a welded lap joint is a function of the plates 'overlap width'. Duflo et al. [3] studied the interaction of Lamb waves with defects on bonded composite plates. It was observed that some frequency components of the incident signal have greater attenuation. The flaw was evaluated using attenuated frequency components of the Lamb mode. Koh et al. [6,7] modelled the impact damage by using a local change in the stiffness of the structural material and by introducing a delamination. Effects on the transmission of the incident Lamb wave when it propagated through a region of changing density were analysed. It was found that the change in local stiffness had a large impact on attenuation of the incident Lamb wave. It was observed that, for a given change in local stiffness, the apparent size of this lower stiffness region has a limited effect on the attenuation of the incident Lamb wave. Koh et al. [6] showed that disbond growth can be predicted by the attenuation of the transmitted Lamb wave power. These results agree with the experimental results reported elsewhere. Tan et al. [17] showed experimentally that Lamb waves are sensitive to delaminations in a solid graphite-fiber composite plate. Change in amplitude (attenuation) was used as a criterion for assessing the delamination size. Toyama and Takatsubo [18] proposed a quick and quantitative inspection technique for impact-induced delamination in composite laminates. This method detects and identifies the delamination, and evaluates its size and location using Lamb wave features such as velocity and attenuation.

Attenuation of a guided wave is an important facet in tomographic reconstruction [15]. Prasad et al. [13] observed that attenuation of a Lamb mode when it encountered a defect was evident in the tomogram where the defective region is clearly distinguishable from the rest of the plate. The intensity around the defective region was found to be less when compared with its surroundings. It was concluded that the tomogram depicts the distribution of the Lamb

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wave attenuation as it traverses through the plate. Tomographic imaging algorithms normally require measurements of either attenuation or ToF be made along a series of ray-paths. Measurements of changes in ToF yielded images of slowness changes (slowness is the inverse of velocity) while measurements of amplitude changes resulted in images of changes in the attenuation parameter [4].

In the case of fiber reinforced composites, one of the main complexities involved in the use of guided waves is attenuation. This is caused by the viscoelastic nature of the resin and also due to scattering that occurs at higher frequencies when fibers and other heterogeneities are encountered. The fundamental symmetric mode (S_o) has lower attenuation than the fundamental anti-symmetric mode (A_o) while propagating through viscoelastic material such as composites [20,12]. The anti-symmetric modes propagate with minimal attenuation in metal plates; however, in perspex and composites, the attenuation of anti-symmetric modes is much higher than in symmetric modes [10]. There are two reasons for the attenuation of Lamb waves: (a) geometry and (b) material. Attenuation due to geometry occurs when Lamb wave propagates through damaged regions and/or encounter the plate edges. In this type of attenuation, there is no dissipation of energy. Geometrical attenuation of Lamb waves is used for NDE/SHM applications. Lamb wave attenuation due to material is inherently present in the propagating medium such as composites, polymers and perspex. In general, when a guided wave propagates through a defective region in a viscoelastic material, the reduction in amplitude is due to both geometry and the properties of material/medium. Attenuation due to material (or material attenuation) can be measured experimentally. This is expressed in terms of attenuation coefficient expressed in Nepers per meter (Np/m). Castaings and Hosten [2] measured the attenuation of Lamb waves, employing air-coupled ultrasonic transducers, in isotropic and viscoelastic (composite) materials. Ning et al. [11] while proposing a technique for identifying the location of a delamination in a cross-ply laminated composite using the S_o mode, determined the attenuation coefficients of a Carbon Fiber Reinforced Plastic (CFRP) material by matching the amplitudes of wave groups in a numerical model.

Numerical modelling of guided wave propagation helps immensely in understanding the interaction phenomenon between the material damage and the guided wave. The Finite Element Analysis (FEA) approach is traditionally used for modelling and simulation of elastic wave propagation. Many authors modelled Lamb wave propagation through composite media without considering the effects of material attenuation. Lamb wave parameters such as group velocity, phase velocity, etc. obtained from numerical simulations matched well with the theoretical values. In numerical simulations, geometrical attenuation of Lamb waves can be captured effectively, if all the geometrical features are included in the numerical model. Lamb wave attenuation due to material, however, is an important aspect from NDE/SHM. Waves undergo considerable attenuation when they propagate through composite and polymer materials [16]. In the FE model, if material attenuation is also incorporated, then NDE/SHM methods/techniques based on the change in amplitude (attenuation) can be understood better and implemented effectively.

In vibration analysis, the damping that is often used in the mathematical model for simulation of a dynamic response of a structure is proportional to the stiffness and mass density of the structure. This is referred to as Rayleigh damping [9]. This damping model is implemented in many computer programs (e. g ANSYS, ABAQUS, COMSOL, etc.) in order to obtain results for numerically sensitive structural systems. An important part of Rayleigh damping is determination of mass and stiffness proportionality constants, α and β, respectively. These two parameters must be used as inputs in a numerical model to incorporate Rayleigh damping in vibration analysis.

In the present work, a Rayleigh (proportional) damping model is used to study the material attenuation of Lamb waves. The proportionality constants are expressed in terms of the attenuation coefficient, group velocity and central frequency of excitation. Thus estimated, proportionality constants are thereafter used in numerical modelling to capture the material attenuation of Lamb waves.

The organization of this paper is as follows. Equations relating to Lamb wave attenuation constants derived with relation to group velocity, attenuation coefficient and excitation frequency are presented in Section 2. Numerical and experimental validations carried out on the proposed Lamb wave attenuation constants are explained in Section 3. Results and discussion, and conclusions are presented in Sections 4 and 5, respectively.

2. Expressions for Lamb wave attenuation constants

In vibration analysis, a proportional damping model is widely used to capture the damping behavior of vibrating structures. The popularity of this model is due to its simplicity in implementation, de-coupling of the governing equations and control over the tuning of mass and stiffness proportional constants, α and β, respectively. Many FE analysis codes implement the Rayleigh damping model. The following equation relates the coefficient of damping to mass and stiffness through proportionality constants [9].

$$c = \alpha m + \beta k \tag{1}$$

The above equation can also be expressed in terms of the damping ratio, ζ,

$$\zeta = \frac{c}{c_c} = \frac{1}{2}\left(\frac{\alpha}{\omega} + \beta\omega\right) \tag{2}$$

where c_c is the coefficient of critical damping and ω is the circular frequency = $2\pi f$ and f is the frequency.

The logarithmic decrement [9] is expressed as follows.

$$\ln\left(\frac{u}{u_n}\right) = \frac{2\pi\zeta n}{\sqrt{1-\zeta^2}} \tag{3}$$

where u is amplitude and u_n is the amplitude of vibration after n cycles. Since ζ is a very small number, its square is much less than unity. So, the denominator in Eq. (3) is assumed to be unity. Eq. (3) then becomes

$$r_v = \ln\left(\frac{u}{u_n}\right) \approx 2\pi n\zeta \tag{4}$$

When the frequency is high, vibration, which is a global phenomenon, becomes a local phenomenon, which is nothing but the wave propagation. Assume that the Lamb wave is propagating in a plate. The amplitudes of Lamb modes are A_1 and A_2 at arbitrary reference positions, x_1 and x_2, respectively. The arrival times, ToF, of Lamb mode are t_1 and t_2 at reference positions x_1 and x_2, respectively. If C_g is the group velocity, then the following expression represents the ToF, C_g and distance of travel, $\Delta x = x_2 - x_1$.

$$\Delta t = t_2 - t_1 = \frac{\Delta x}{C_g} \tag{5}$$

If k_i is the attenuation coefficient of the viscoelastic medium, the following equation can be written for obtaining the amplitude ratio [2,16].

$$\frac{A_1}{A_2} = \exp(-k_i(x_1 - x_2)) = \exp(k_i\Delta x)$$

$$r_w = \ln\left(\frac{A_1}{A_2}\right) = k_i\Delta x \tag{6}$$

Assuming that the locations of reference positions were chosen in such a way that Δt can be expressed as the product of the num-