Composite Structures 93 (2011) 2120–2125

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Dynamic behaviour of sandwich structure containing spring-mass resonators

J.S. Chen, B. Sharma, C.T. Sun *

School of Aeronautics and Astronautics, Purdue University, Armstrong Hall, West Lafayette, IN 47907, USA

ARTICLE INFO

Keywords:
Sandwich beam
Resonator
Negative effective mass
Wave propagation
Attenuation

ABSTRACT

The behaviour of wave propagation in a sandwich beam with internal resonators is studied analytically and experimentally. It is found that, with the occurrence of a bandgap, waves generated by actuations in certain frequency range are forbidden to propagate into the sandwich beam without attenuation. By tailoring the local resonance frequency of the resonator, the range and the location of the bandgap can be selected. Also, from the experimental and FEA results, it is seen that waves can be significantly attenuated in a sandwich beam with a finite number of internal resonators if the forcing frequency is near the local resonance frequency. It is concluded that the resonators can be used as a mechanism for suppressing the flexural motion in a sandwich beam.

1. Introduction

The blast resistance of sandwich structures has attracted much interest among structural engineers. Many researchers have focused on adjusting the core properties, such as density, thickness, and mechanical properties to enhance the impact resistant strength on the sandwich structure [1–4]. Recently, acoustic metamaterials have presented an additional possibility in stopping waves from propagation into the material. In general, metamaterials are regarded as materials with manmade microstructures exhibiting behaviour that is not found in natural materials. For instance, Liu et al. [5] investigated a type of composite fabricated using resonators in the form of lead balls coated with silicon rubber, which are embedded in an epoxy matrix. A number of other authors [6–10] considered lattice systems consisting of simple spring-mass resonators. It was found that, near the local resonance frequency of the resonator, the metamaterial behaves as a material with negative effective mass density. Such an anomalous response of the system is the consequence of the coupling of the long-wavelength elastic wave in the host medium with the localized vibration motion of the local resonators. It was found that metamaterials possessing this unusual property can give rise to a significant wave attenuation effect near the local resonance frequency. By tailoring the local resonance frequency, the design of filtering low-frequency stress waves can be achieved. Recently, Pai [11] studied an elastic wave absorber consisting of a uniform elastic bar with many small spring-mass subsystems attached to the bar at separate longitudinal locations. From the finite-element simulations, it was found that this type of wave absorber has a similar effect of conventional mechanical vibration absorbers, which use the local resonance of subsystems to generate inertia forces to work against the external load and prevent elastic waves from propagating in the bar.

In this study, we continue the work initiated by Sun and Chen [12] who studied the dynamic behaviour of a sandwich beam containing resonators embedded in the core. In addition to numerical simulations, experimental results are also presented. Moreover, the effect of spring constant and internal mass of resonators on the extent of bandgap is investigated.

2. Investigation of bandgap

Fig. 1 shows the schematic of the sandwich construction with a core that contains resonators. To simplify the dynamic analyses of the sandwich beam with resonators, we distribute the resonators uniformly over the entire length of the beam using a volume-averaging process. By so doing, the sandwich beam with resonators can be described by continuous field variables and the equations of motion can be written in terms of partial differential equations. A representative sandwich beam element (a unit cell) is taken as shown in Fig. 2a. Each resonator is composed of a mass m_1 and two linear springs with a spring constant denoted by k_1. From an alternative perspective, the unit cell may be viewed as the combination of an equivalent sandwich beam element connected to a mass with a spring, shown in Fig. 2b. The effective core yields the combined stiffness properties of the core and the casing.

The effective sandwich beam is modelled as a Timoshenko beam having the bending rigidity *EI* and shear rigidity *GA* that are equivalent to the original sandwich beam but without resonators. Based on an energy averaging technique and the Hamilton's principle, equations of motions have been derived and employed

* Corresponding author. Tel.: +1 765 494 5130; fax: +1 765 494 0307.
E-mail address: sun@purdue.edu (C.T. Sun).

0263-8223/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2011.02.007

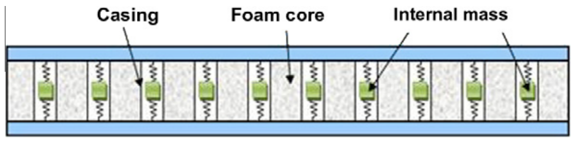

Fig. 1. Sandwich beam with internal resonators.

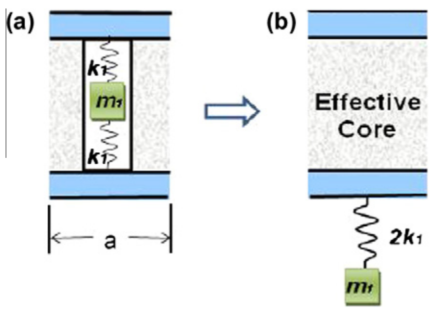

Fig. 2. Schematic diagram of (a) a unit cell and (b) its equivalent model.

Table 1
Dimensions and material constants of specimen.

E_f (GPa)	h_f (m)	h_c (m)	b (m)	a (m)	k_1 (N/m)	m_1 (kg)
97.19	7.62×10^{-4}	3.02×10^{-2}	0.019	0.012	5415.74	1.17×10^{-3}

Table 2
Effective properties of the sandwich beam.

EI (Pa m^4)	GA (Pa m^2)	ρA (kg/m)	ρI (kg m)
677	1.12×10^4	0.1248	1.69×10^{-5}

to study wave propagation [12]. Unlike a conventional Timoshenko beam, there are three coupled equations of motions which can be expressed as:

$$\begin{aligned} &GA(v'' + \varphi') - \rho A \ddot{v} + \frac{2k_1}{a}(v_1 - v) = 0 \\ &EI\varphi'' - GA(v' + \varphi) - \rho I \ddot{\varphi} = 0 \\ &\frac{2k_1}{a}(v_1 - v) + \frac{m_1}{a}\ddot{v}_1 = 0 \end{aligned} \tag{1}$$

where GA is transverse shear rigidity, EI is bending rigidity, ρA is mass of the sandwich beam per unit length, ρI is rotary inertia, v is the transverse displacement of the sandwich beam, φ is the rotation of the cross-section about the y-axis, v_1 is the transverse displacement of the internal mass m_1, and a is the spacing of the resonators. By assuming the three kinematic variables, v, φ, and v_1, to be in the form of $e^{i(qx-\omega t)}$ for plane harmonic wave propagation, the dispersion equation can be obtained from Eq. (1). We have,

$$A_1\omega^6 + A_2\omega^4 + A_3\omega^2 + A_4 = 0 \tag{2}$$

where

$$A_1 = -\rho I \rho A\left(\frac{m_1}{a}\right),$$

$$A_2 = \left\{(\rho IGA + EI\rho A)\left(\frac{m_1}{a}\right)q^2 + GA\rho A\left(\frac{m_1}{a}\right) + \rho I\rho A\left(\frac{2k_1}{a}\right) + \left(\frac{2k_1m_1}{a^2}\right)\rho I\right\},$$

$$A_3 = -\left\{EIGA\left(\frac{m_1}{a}\right)q^4 + \left\{(\rho IGA + EI\rho A)\left(\frac{2k_1}{a}\right) + \left(\frac{2k_1m_1}{a^2}\right)EI\right\}q^2 + GA\rho A\left(\frac{2k_1}{a}\right) + \left(\frac{2k_1m_1}{a^2}\right)GA\right\},$$

$$A_4 = EIGA\left(\frac{2k_1}{a}\right)q^4.$$

By solving Eq. (2) for frequency ω for varying wave number q, the dispersion curves are obtained.

The dimensions and material constants adopted in the computation are listed in Table 1 and the corresponding sandwich beam properties are given in Table 2. These parameters are chosen based on the sandwich specimen used in the experiment presented in Section 4. The face sheets are laminates AS43501-6 $[0_2/90_2/0_2]$ with an effective elastic Young's modulus E_f; the core is Rohacell IG-51 foam; and a plastic casing is used to contain the resonator. The height of the casing is the same as the thickness of the foam core. The effective shear rigidity of the sandwich beam with the Rohacell core and embedded plastic casings was evaluated from the standard three point bending test [13]. The thickness of the face sheet and that of the core are denoted by h_f and h_c, respectively, and the width of the beam is denoted b. The densities of the face sheet, the core, and the casing are 1550 kg/m^3, 52.1 kg/m^3, and 2090 kg/m^3, respectively. The outer diameter and the thickness of the casing are 0.00635 m and 0.00081 m, respectively.

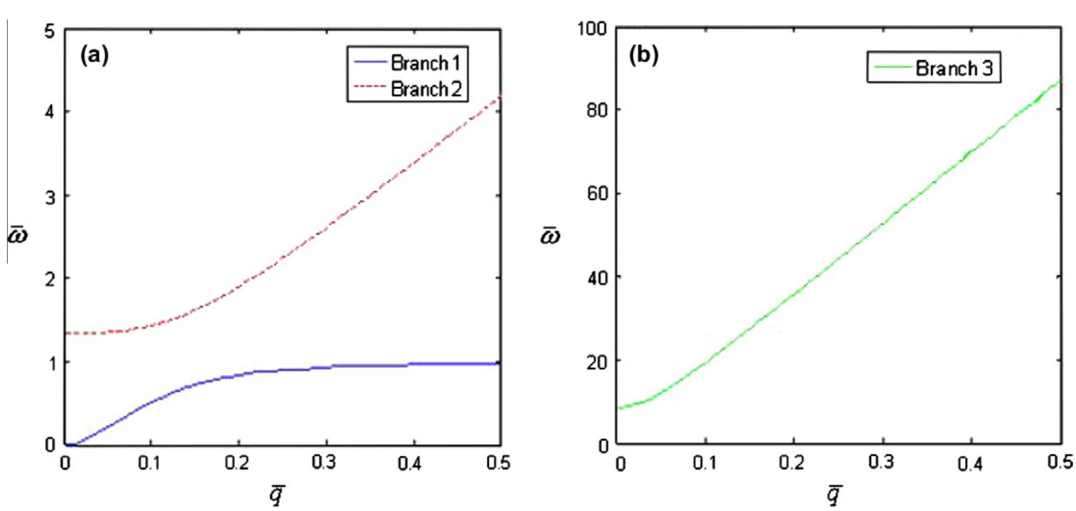

Fig. 3. Dispersion curves for harmonic waves: (a) lowest two branches and (b) the highest branch.