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Explicit local buckling analysis of rotationally-restrained orthotropic plates under uniform shear

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ABSTRACT

The explicit closed-form local buckling solution of in-plane shear-loaded orthotropic plates with two opposite edges simply supported and other two opposite edges either both rotationally restrained or one rotationally restrained and the other free is presented. Based on the boundary condition of the other two opposite edges, two types of plates are considered: the RR (both the edges rotationally restrained) and RF (one edge restrained and the other free) plate elements. Different plate buckled shape functions are proposed, and the approximate explicit expressions for the buckling loads are derived using the Rayleigh–Ritz method for the plate with the generic rotationally-restrained (R) boundary conditions which can be reduced to two extreme cases, i.e., simply supported (S) and clamped (C). The accuracy of the derived explicit solutions is verified by comparing the predictions with the existing solutions and numerical finite element analysis, and excellent agreements are obtained. The effects of material and boundary restraining parameters on the local shear buckling behavior of the plate elements are discussed. The derived explicit formulas for the shear buckling loads are straightforward, efficient and reliable for preliminary engineering design and analysis of composite structures under primarily shear-dominant loading conditions.

1. Introduction

Advanced composite structures commonly in thin-walled configurations and made of fiber-reinforced plastic (FRP) laminates are being increasingly used in structural applications. When undergoing large deflection and strains, unlike the conventional materials that yield (steel) or crack (concrete), the composite materials usually exhibit relatively linearly elastic behavior because of their relatively low stiffness and high strength (e.g., E-glass fiber-reinforced composites). Therefore, buckling is a major concern for thin-walled FRP structures in design, and the critical buckling load is directly related to the load-carrying capacity of such structures. One of the simple methods to deal with the local buckling problem of thin-walled FRP structures is the discrete plate analysis technique [1]. In this technique, each component (e.g., flanges or webs) in the cross-section of composite FRP shapes is modeled as a thin plate under the restraints from the conjunctions of webs and flanges. The restraints along the edges of the plate elements are usually modeled as the rotational restraining springs, and the spring stiffness can be adjusted according to the joint (conjunction) stiffness from the adjacent plates. For instance, the null value of the rotational restraining stiffness corresponds to a simply-supported boundary condition (S); while the rotational restraining stiffness of infinite large is treated as a clamped condition (C) along the plate edges. In reality, the rotational restraining stiffness is somewhere between the two extreme cases of S and C.

A large amount of studies in local buckling analysis has been conducted in general. The main results of these studies have been reviewed and included in a number of text books [2,3]. Most of the efforts have been devoted to the laminated plates under uniaxial compressive loads, whereas the shear-controlled local buckling has only attracted limited attention. The shear buckling problem does not readily yield a reasonably exact theoretical solution; thus, the effort is directed to solve the problem approximately on a rational basis.

In recent two decades, different analytical and numerical tools, such as the Rayleigh–Ritz [4–11], Galerkin [12], finite element [13–15], and finite strip [16,17] methods, have been successfully used to solve the buckling problem of plates under in-plane shear loading. Based on the Rayleigh–Ritz method, the approximate formulas for the buckling of angle-ply and cross-ply laminated plates subjected to combined edge compression and shear with the four edges simply supported were presented by Kumar and Kishore [4]. Using the sine functions up to an order of eight for summation as the shape functions, both the symmetric and antisymmetric buckling modes were discussed. Numerical solutions for the local buckling of orthotropic plates under uniform compression, linearly varying compression or shear load were obtained by Tarjan et al. [11] using the Rayleigh–Ritz method. The displacement was

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simulated by a trigonometric series with 71 terms and 41 terms in the width and longitudinal directions, respectively. The explicit expressions for local buckling loads of orthotropic plates with rotationally-restrained edges were obtained by curve-fitting the numerical solutions. The Rayleigh–Ritz method was also adopted by other researchers to solve the buckling problem using different shape functions, e.g., the pb-2 functions [7,9,10].

The problem of buckling of thin rectangular anisotropic plates subjected to uniform in-plane shear load, as well as some other in-plane loads, with all edges clamped was studied by Kosteletos [12] using the Galerkin's method and adopting the limit terms of the transcendental functions to simulate the buckled shape. With the same method, the buckling and postbuckling analyses of perfect and simplified imperfect structures were evaluated by Zaras et al. [5], and the study was restricted to the simply supported boundary condition.

An extensive treatment of the problem under discussion was offered by Qatu and Leissa [13], who elaborated the conditions for the bifurcation buckling to occur, e.g., the type of laminates, boundary conditions, and in-plane loading, using the numerical finite element method. The lock-free element was developed by Singh et al. [14] to investigate the buckling behavior of the moderately-thick laminated plates subjected to various in-plane edge loads. The finite strip method was adopted by Loughlan [17] to examine the effect of bending-twisting coupling on the shear buckling behavior of laminated composite structures.

Using the energy method, the present study aims to investigate the stability of orthotropic thin plates with two opposite edges simply supported and the other two opposite edges either both rotationally restrained (RR) or one edge rotationally restrained and the other one free (RF) under the uniformly distributed in-plane shear load along the four edges as shown in Figs. 1–3. The explicit closed-form solutions are in need for the preliminary design and analysis. According to the authors' knowledge, there are no explicit closed-form buckling solutions for the thin laminated composite plates with either the RR or RF boundary conditions under pure shear. Either the trigonometric or polynomial series is occasionally adopted in the literature as the approximate buckling shape functions. Though the accuracy is improved if more series expansion terms are included, the high order eigenvalue problems are tedious to be solved explicitly. In this study, different shape functions are considered to obtain the shear buckling load of the elastically-restrained plates explicitly, and the validity of the solution is confirmed by comparing the predictions with the finite element analysis and other numerical solutions available in literature.

2. Generalized buckling analysis

For relatively long plates (i.e., $\gamma = a/b > 3$), the effect of boundary conditions at the shorter edges on the critical buckling load is negligible [7]. Therefore, the boundary conditions of long plates can be assumed as identical to a periodic plate element, given the constant material properties and applied loads. It is thus possible to extract a representative plate element (as shown in Fig. 3) employing the periodic geometric properties and conditions.

In the case of thin-walled beams subjected to the transverse load, the flanges can be assumed under the in-plane axial load, while the web primarily carries the shear load and can be modeled as a plate rotationally-restrained at the web-flange intersections and subjected to in-plane shear load. While for the stiffened composite panels under in-plane shear load, the plate element between the stiffeners can be simulated as a rotationally-restrained plate under pure shear. The segments considered in this work are thin orthotropic rectangular plates taken from the web or panel with dimensions of $a \times b$. As shown in Fig. 1, the origin of the coordinates lies at the left corner of the plate. Two types of rotationally-restrained plate elements (i.e., the RR and RF plates) from different beam cross sections (e.g., I- and T-sections) or stiffened composite panels are investigated. The transverse edges ($x = 0, a$) are assumed simply supported. The other two longitudinal boundaries ($y = 0, b$) of the RR plate are restrained against rotation, while the RF plate has one edge rotationally restrained and the other edge free. Only the uniform in-plane shear loads are applied to both the plates. Bifurcation buckling is considered in this study, and the pre-buckling and post-buckling states are not included.

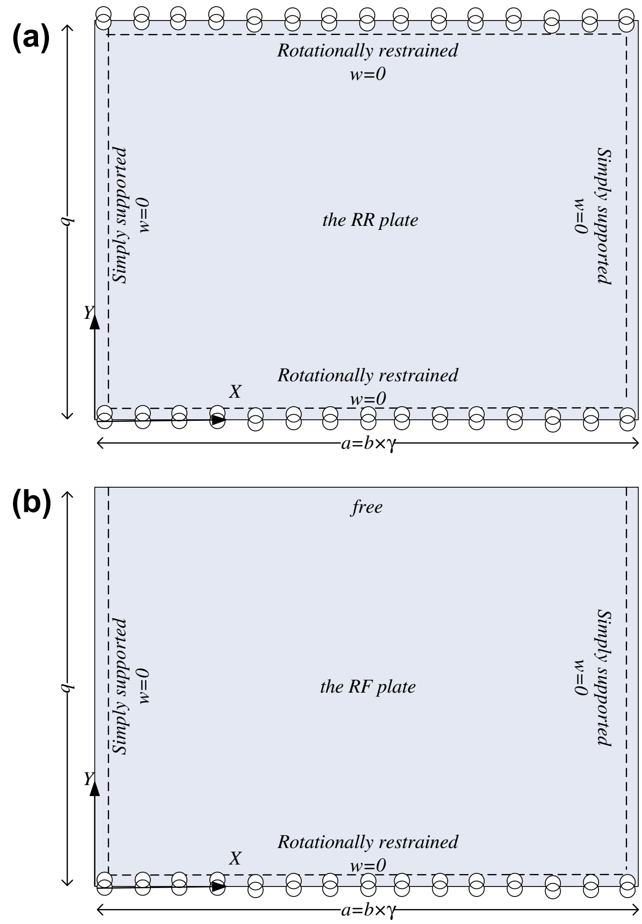

Fig. 1. The boundary conditions of two types of plates: (a) the RR plate, and (b) the RF plate.

The restraint of the plates against edge rotation is modeled as rotational springs, and it is controlled by the spring stiffness. For the RR plate, the possible edge boundary condition scenarios along $y = 0, b$ are totally rotationally free (SS), fully rotationally clamped (CC) or partially restrained against rotation (RR). While for the case of RF plate, only one edge $y = 0$ can be adjusted to the three corresponding types of edge conditions (i.e., S, C, and R).

The most convenient method to obtain the buckling load of the plate is the energy method. The energy (Π) associated with the plate buckled in an assumed shape is composed of three parts: the strain energy (U_e) of the plate, the restraining energy (U_s) stored in the spring system along the plate edge, and the work (W) done by the applied shear load, and they can be expressed, respectively, as

$$U_e = \frac{1}{2}\iint_{\Omega}\left[D_{11}^2\left(\frac{\partial^2 w}{\partial x^2}\right) + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]dxdy \quad (1)$$