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Natural frequencies of cracked functionally graded material plates by the extended finite element method

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ABSTRACT

In this paper, the linear free flexural vibration of cracked functionally graded material plates is studied using the extended finite element method. A 4-noded quadrilateral plate bending element based on field and edge consistency requirement with 20 degrees of freedom per element is used for this study. The natural frequencies and mode shapes of simply supported and clamped square and rectangular plates are computed as a function of gradient index, crack length, crack orientation and crack location. The effect of thickness and influence of multiple cracks is also studied.

1. Introduction

Engineered materials such as laminated composites are widely used in automotive and aerospace industry due to their excellent strength-to and stiffness-to-weight ratios and their possibility of tailoring their properties in optimizing their structural response. But due to sudden change in material properties between the layers in laminated composites, these materials suffer from premature failure or by the decay of stiffness characteristics because of delaminations and chemically unstable matrix and lamina adhesives. The emergence of functionally graded materials (FGMs) [1,2] has revolutionized the aerospace and aircraft industry. The FGMs used initially as thermal barrier materials for aerospace structural applications and fusion reactors are now developed for general use as structural components in high temperature environments. FGMs are manufactured by combining metals and ceramics. These materials are inhomogeneous, in the sense that the material properties vary smoothly and continuously in one or more directions. FGMs combine the best properties of metals and ceramics and are strongly considered as a potential structural material candidates for certain class of aerospace structures exposed to a high temperature environment.

It is seen from the literature that the amount of work carried out on the vibration characteristics of FGMs is considerable [3–9]. He et al. [5] presented finite element formulation based on thin plate theory for the shape and previous vibration control of FGM plate with integrated piezoelectric sensors and actuators under mechanical load whereas Liew et al. [6] have analyzed the active previous vibration control of plate subjected to a thermal gradient using shear deformation theory. Ng et al. [7] have investigated the parametric resonance of plates based on Hamilton's principle and the assumed mode technique. Yang and Shen [10] have analyzed the dynamic response of thin FGM plates subjected to impulsive loads using Galerkin procedure coupled with modal superposition method whereas, by neglecting the heat conduction effect, such plates and panels in thermal environments have been examined based on shear deformation with temperature dependent material properties in [8]. Qian et al. [11] studied the static deformation and vibration of FGM plates based on higher-order shear deformation theory using meshless local Petrov–Galerkin method. Matsunaga [4] presented analytical solutions for simply supported rectangular FGM plates based on second-order shear deformation plates. Vel and Batra [3] proposed three-dimensional solutions for vibrations of simply supported rectangular plates. Reddy [12] presented a finite element solution for the dynamic analysis of a FGM plate and Ferreira et al. [9] peformed dynamic analysis of FGM plate based on higher-order shear and normal deformable plate theory using the meshless local Petrov–Galerkin method. Akbari et al. [13] studied two-dimensional wave propagation in functionally graded solids using the meshless local Petrov–Galerkin method. The above list is no way comprehensive and interested readers are referred to the literature.

FGM plates or in general plate structures, may develop flaws during manufacturing or after they have been subjected to large cyclic loading. Hence it is important to understand the dynamic response of a FGM plate with an internal flaw. It is known that cracks or local defects affect the dynamic response of a structural

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member. This is because, the presence of the crack introduces local flexibility and anisotropy. Moreover the crack will open and close depending on the vibration amplitude. The vibration of cracked plates was studied as early as 1969 by Lynn and Kumbasar [14] who used a Green's function approach. Later, in 1972, Stahl and Keer [15] studied the vibration of cracked rectangular plates using elasticity methods. The other numerical methods that are used to study the dynamic response and instability of plates with cracks or local defects are: (1) finite fourier series transform [16]; (2) Rayleigh–Ritz Method [17]; (3) harmonic balance method [18]; and (4) finite element method [19,20]. Recently, Huang et al. [21] proposed solutions for the virbation of side-cracked FGM thick plates based on Reddy third-order shear deformation theory using Ritz technique. Kitipornchai et al. [22] studied nonlinear vibration of edge cracked functionally graded Timoshenko beams using Ritz method. Yang et al. [23] studied the nonlinear dynamic response of a functionally graded plate with a through-width crack based on Reddy's third-order shear deformation theory using a Galerkin method.

In this paper, we apply the extended finite element method (XFEM) to study the free flexural vibrations of cracked FGM plates based on first order shear deformation theory. We carry out a parametric study on the influence of gradient index, crack location, crack length, crack orientation and thickness on the natural frequencies of FGM plates using the 4-noded shear flexible element based on field and edge consistency approach [24]. The effect of boundary conditions and multiple cracks is also studied. Earlier, the XFEM has been applied to study the vibration of cracked isotropic plates [25–27]. Their study focussed on center and edge cracks with simply supported and clamped boundary conditions.

The paper is organized as follows, the next section will give an introduction to FGM and a brief overview of Reissner–Mindlin plate theory. Section 3 illustrates the basic idea of XFEM as applicable to plates. Section 4 presents results for the free flexural vibration of cracked plates under different boundary conditions and aspect ratios, followed by concluding remarks in Section 5.

2. Theoretical formulation

2.1. Functionally graded material

A functionally graded material (FGM) rectangular plate (length a, width b and thickness h), made by mixing two distinct material phases: a metal and ceramic is considered with coordinates x, y along the in-plane directions and z along the thickness direction (see Fig. 2). The material on the top surface ($z = h/2$) of the plate is ceramic and is graded to metal at the bottom surface of the plate ($z = -h/2$) by a power law distribution. The homogenized material properties are computed using the Mori–Tanaka Scheme [28,29].

2.1.1. Estimation of mechanical and thermal properties

Based on the Mori–Tanaka homogenization method, the effective bulk modulus K and shear modulus G of the FGM are evaluated as [28–30,11]

$$\begin{aligned} \frac{K-K_m}{K_c-K_m} &= \frac{V_c}{1+(1-V_c)\frac{3(K_c-K_m)}{3K_m+4G_m}} \\ \frac{G-G_m}{G_c-G_m} &= \frac{V_c}{1+(1-V_c)\frac{(G_c-G_m)}{G_m+f_1}} \end{aligned} \tag{1}$$

where

$$f_1 = \frac{G_m(9K_m+8G_m)}{6(K_m+2G_m)} \tag{2}$$

Here, $V_i (i = c, m)$ is the volume fraction of the phase material. The subscripts c and m refer to the ceramic and metal phases, respectively. The volume fractions of the ceramic and metal phases are related by $V_c + V_m = 1$, and V_c is expressed as

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^n, \quad n \geqslant 0 \tag{3}$$

where n in Eq. (3) is the volume fraction exponent, also referred to as the gradient index. Fig. 1 shows the variation of the volume fractions of ceramic and metal, respectively, in the thickness direction z for the FGM plate. The top surface is ceramic rich and the bottom surface is metal rich.

The effective Young's modulus E and Poisson's ratio ν can be computed from the following expressions:

$$\begin{aligned} E &= \frac{9KG}{3K+G} \\ \nu &= \frac{3K-2G}{2(3K+G)} \end{aligned} \tag{4}$$

The effective mass density ρ is given by the rule of mixtures as [3]

$$\rho = \rho_c V_c + \rho_m V_m \tag{5}$$

The material properties P that are temperature dependent can be written as [31]

$$P = P_o(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{6}$$

where $P_o, P_{-1}, P_1, P_2, P_3$ are the coefficients of temperature T and are unique to each constituent material phase.

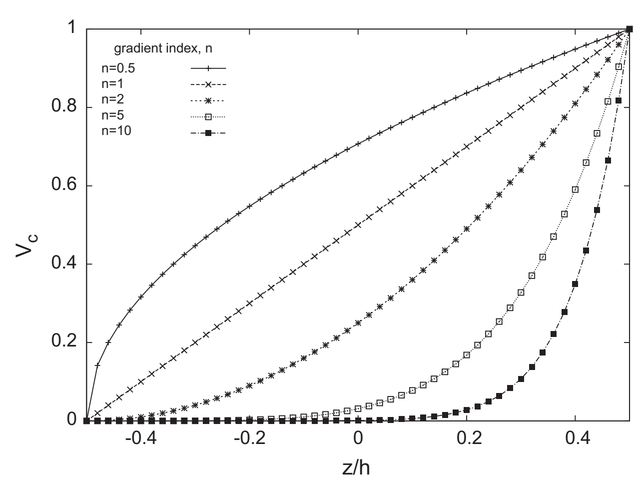

Fig. 1. Through thickness variation of volume fraction.

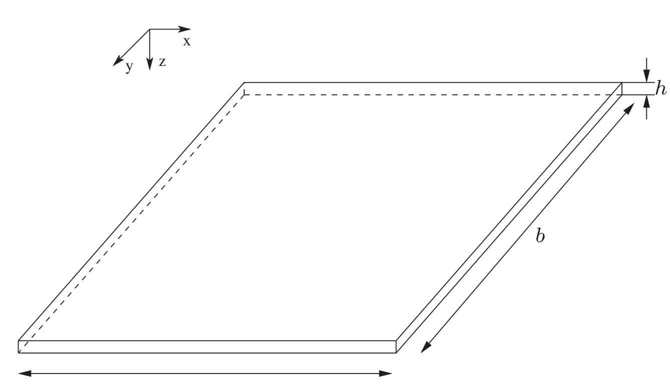

Fig. 2. Co-ordinate system of rectangular plate.