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Free surface and solid boundary effects on the free vibration of cantilevered composite plates

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ABSTRACT

This paper investigates the effects of surrounding boundaries on the free vibration response of fully and partially submerged cantilevered composite plates and how these effects change due to material anisotropy. The results show that added mass significantly reduces the natural frequencies of cantilevered marine structures as a function of relative submergence depth, more so for composite plates than for steel plates because of the much lower ratio of effective structural mass to hydrodynamic added mass. Added mass effects are most dramatic for partially submerged plates as the plates move from being above to completely beneath the free surface. Free surface effects are shown to become negligible for a fully submerged plate parallel to the free surface when the depth of submergence exceeds 50% of the plate length. Solid boundaries are found to have limited effects for fully-submerged plates near a wall, where maximum decreases in resonance frequencies due to increases in added mass are only a few percent.

1. Introduction

In recent years, composite materials are becoming more prevalent in marine applications as an alternative to traditional metallic alloys because they can provide increased strength- and stiffness-to-weight ratios, corrosion resistance, and flexibility. In addition, the anisotropic nature of composite materials allows for elastic tailoring of the coupled bend–twist deformation characteristics to achieve improved hydroelastic performance over a wide range of operating conditions [1–16]. Because of the increased flexibility, reduced mass, and load-dependent deformations, it is critical to understand and to predict the free vibration of these structures to avoid natural or resonant vibrations, and to control the structural and acoustic responses.

Many of the applications for which composite materials have been studied involved cantilevered marine structures such as propeller and turbine blades, sails, rudders, keels, and wings. While composites are commonly used in aerospace applications, and thus their behavior is well-established, much less information is available about the dynamic response of these structures in water. For aerodynamic analyses, fluid inertial (added mass) effects are generally neglected because of the much lower density of air compared to the solid structure. For hydrodynamic analyses, however, added mass cannot be neglected because the fluid inertial force can be of the same order as that of the solid structure, particularly for light composite structures. Further, the added mass of a marine structure depends on its proximity to the free surface and solid boundaries.

Lindholm et al. [17] presented experimental comparisons of the in-air and in-water resonant frequencies of cantilevered steel plates along with analytical plate theory approximations for varying submergence. They showed that the free surface can have a significant effect on the dynamic response of the plate. Similar work has been done for submerged, isotropic, cantilevered plates using various numerical and analytical frameworks [18–26], where in each case, similar conclusions were shown. In general, it was shown that natural frequencies of cantilevered plates decrease with increasing aspect ratio and increase with increasing thickness ratio, both of which are due to changes in the structural stiffness. For submerged plates near the free surface, the in-water resonant frequencies have been shown to decrease with increasing submergence depth due to increasing added mass effects to a point at which the added mass and resulting resonant frequencies become approximately constant, i.e. deep enough such that the effect of the free surface becomes negligible. The reduction in natural frequency is generally shown to be dependent on the mode shape. Haddara and Cao [27] showed similar results for a plate fixed at both ends. Fu and Price [20] noted that free surface effects can produce large variations in the hydrodynamic coefficients for a submerged horizontal plate near the free surface, but that these large variations are not always evident for vertical, surface-piercing plates. Ergin and Ugurlu [22] showed that the generalized added mass matrices are symmetric and that the intermodal coupling terms become stronger with decreasing submergence depths and notes that, near the free surface, added mass effects exhibit frequency dependence

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in low-frequency regions. Similarly, Yadykin et al. [23] shows increased added mass effects with decreasing modes of vibration and with increasing aspect ratios. Further, Rodriguez et al. [24] noted that the difference in frequency between in-air and in-water plates is dependent only on the added mass and not on added damping. For plates near a solid boundary, Brennen [19] noted that the presence of a solid boundary parallel to the plate generally increases the added mass on the plate resulting from fluid accelerations in the region between the plate and the boundary. Korotkin [28] also showed that proximity to a solid boundary perpendicular to cantilevered plate increases the added mass, and hence decreases the resonance frequency, of the plate.

As shown above, experimental and numerical results are available for metallic plates operating above, at, or below the free surface. Such results, however, are not generally available for composite plates. Composite materials, in general, have much lower densities than traditional metallic alloys such as steel, bronze, or aluminum. As a result, composite plates would be expected to be much more sensitive to changes in added mass effects due to proximity to free surface and solid boundaries. Moreover, the anisotropic constitutive behavior of composites will lead to intrinsic bend–twist coupled deformations. Most of the available open literature on the free vibration of composite structures focused on the in-air or in-vacuum response [29–32]. In general, the mode shapes and resonant frequencies of composites depend on the fiber orientation; the results show that the free vibration frequency decreases as the fiber orientation deviates further from the longitudinal direction. Teh and Huang [29] showed that even small changes in fiber angle can have significant effects on the plate modal frequencies and mode shapes. More recently, Kramer et al. [33] presented analytical and numerical investigations of the effect of fiber orientation angle and plate aspect ratio on the in-vacuum and in-water free vibration of cantilevered composite plates. The results show that the ratio of wet-to-dry frequencies are much lower for composite plates than metallic plates due to the much lower structural mass to added mass ratios, and that the frequency ratios of both metallic and composite plates depend on the mode shapes, which are influenced by the fiber orientation angle and plate aspect ratio.

The effects of added mass, material anisotropy, and proximity to the free surface and solid boundary on the free vibration response of composite marine structures must be carefully considered. As such, the objective of this work is to investigate the effects of surrounding boundaries on the free vibration response of a cantilevered composite plate and how these effects change due to material anisotropy.

2. Problem description

This work considers steel and carbon-fiber reinforced polymer (CFRP) cantilevered rectangular plates in proximity to the free surface or a solid boundary. A schematic for a typical cantilever plate is shown in Fig. 1. The dimensions of the plate are defined by its length, width, and thickness, represented by a, b, and t, respectively. Two plates are considered herein, one with an aspect ratio of $\mathcal{AR} = a/b = 2$ and another with an aspect ratio of $\mathcal{AR} = a/b = 5$, corresponding to the plate dimensions from the experimental studies of Lindholm et al. [17]. The dimensions of the two plates are shown in Table 1.

A typical composite plate is constructed of multiple plies in the thickness direction with varying fiber orientations. The authors have previously shown [34,35] that the load-deformation behavior of a multilayer composite laminate can be maintained using an equivalent unidirectional fiber orientation. The principle material axes of the CFRP plate are denoted as the 1–2 axes, where the 1-direction coincides with the equivalent fiber angle and lies at an angle θ with respect to the x–y axes, as shown in Fig. 1.

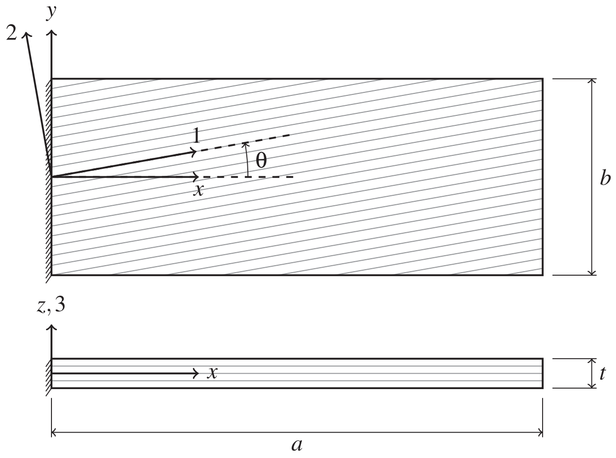

Fig. 1. Diagram of a typical rectangular cantilever beam.

Table 1
Plate dimensions.

Item	$\mathcal{AR} = 2$, $b/t = 76.4$ (m)	$\mathcal{AR} = 5$, $b/t = 42.0$ (m)
a	0.4026	1.0160
b	0.2032	0.2032
t	0.00266	0.00484

The steel plate is modeled as a linear elastic, isotropic material. The CFRP plate is modeled as a linear elastic, orthotropic material, where the 1-direction is associated with the fiber direction, and the stiffnesses in the 2- and 3-directions are assumed to be equal. The assumed material behavior is specified using five material properties, E_1, E_2, G_{12}, ν_{12}, and ν_{23}, where E_i is the Young's modulus in the ith direction and G_{ij} and ν_{ij} are the shear modulus and Poisson's ratio in the i–j plane, respectively. The assumed material parameters for the steel, CFRP, water, and air are shown in Table 2.

3. Methodology

The response of the submerged plate is formulated within a 3-D finite element domain using the commercial finite element solver ABAQUS/Standard [36]. The fluid domain is modeled using 3-D,

Table 2
Material properties for the steel, CFRP, water, and air.

Material	Parameter	Symbol	Value	Unit
Steel	Solid density	ρ_s	7827	kg/m^3
	Young's modulus	E	207	GPa
	Poisson's ratio	ν	0.30	–
CFRP	Solid density	ρ_s	1500	kg/m^3
	Young's modulus (fiber direction)	E_1	171	GPa
	Young's modulus (orthogonal direction)	E_2	9.08	GPa
	Shear modulus	G_{12}	5.29	GPa
	Poisson's ratio	ν_{12}	0.32	–
	Poisson's ratio	ν_{23}	0.29	–
Water	Density	ρ_w	1000	kg/m^3
	Bulk modulus	$(K_f)_w$	2.20	GPa
Air	Density	ρ_a	1.225	kg/m^3
	Bulk modulus	$(K_f)_a$	142	kPa