Composite Structures 96 (2013) 670–682

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Three dimensional shear buckling of FG plates with various boundary conditions

B. Uymaz [a,*], M. Aydogdu [b]

[a] Namık Kemal University, Department of Mechanical Engineering,Corlu, Tekirdag, Turkey
[b] Trakya University, Department of Mechanical Engineering, Edirne, Turkey

ARTICLE INFO

Article history:
Available online 25 August 2012

Keywords:
Functionally graded plate
Shear buckling
Ritz method
Mode shape

ABSTRACT

In this paper, the buckling of functionally graded plates under shear are considered for various boundary conditions. The buckling results obtained by using Ritz method based on the three-dimensional linear elasticity theory and the higher order shear deformation theories. The material properties of the plate vary through the thickness direction according to a simple power law. The comparison results are presented and the variation of the critical shear buckling load and the critical shear buckling modes with the shear buckling characteristics like the material composition, the plate geometry parameters (side-side ratio, side-thickness ratio) are investigated.

1. Introduction

Functionally graded materials (FGMs) are made of advanced composites in which the material properties and thus the mechanical properties vary smoothly and continuously from one surface to the other. These special characteristics make the preferable to conventional composite materials, which are subject to delamination, for engineering applications [1].

Because of increasing use of these materials for structural components in many engineering applications, the analyse of mechanical behavior of these materials under mechanical and thermal loads gains importance among researchers. Some of researchers analyzed the buckling problem including thermal loads based on the first order shear deformation theory [2,3], third order shear deformation theory [4–6], three-dimensional elasticity theory by using the finite element method [7,8] and closed-form solutions [9].

In addition some of researchers discussed the buckling problem including axial buckling loads and also including Ritz method with different admissible functions. Dickinson and Di Blasio [10] illustrated the applicability of the Gram–Schmidt generated polynomials in the Rayleigh–Ritz method to the analysis of orthotropic, in-plane loaded plates. Uymaz and Aydogdu [11,12] investigated free vibration and axial buckling problem based on three-dimensional elasticity theory and Uymaz et al. [13] investigated vibration of functionally graded (FG) plates based on and five-degree-of-freedom theory by using the Ritz method with Chebyshev displacement functions.

Aydogdu [14], and Taskin et al. [15] presented buckling of functionally graded plates under in-plane loads using the classical plate theory in the Navier method. Abrate [16] considered the problems of free vibrations, buckling and static deflections of functionally graded plates. Bodaghi and Saidi [17] investigated an analytical method for buckling analysis of functionally graded plates by using the boundary layer function. Thai and Choi [18] developed an efficient and simple refined theory for buckling analysis of functionally graded plates. Ferreira et al. [19] employed a meshless method to analyze the static deformation of a simply supported functionally graded plate using third-order shear deformation theory.

Different from those, Aydogdu [20], considered vibration and buckling of axially functionally graded simply supported beams by using the semi-inverse method and Civalek [21] employed a numerical solution of free vibration, static and buckling analyses of rectangular thick plate via the DSC approach.

Traditionally, the buckling analyses under mechanical loads are performed for axial loading types including compression and tension. However, the plates as a structural components are subjected to axial loads and shear loads. To the author's knowledge there is no study of the three-dimensional shear buckling of functionally graded plates. Some few authors studied shear buckling problem. Southwell and Skan Sylvia [22] and Lekhnitskii [23] solved the shear buckling problem for isotropic and orthotropic plates, respectively. Budiansky and Connor [24] considered this problem for clamped boundary conditions. Azikov et al. [25] investigated combined in-plane compression and shear and Lopatin [26] considered combined compression and in-plane bending for orthotropic plates. Lopatin and Korbut [27] considered the buckling problem for clamped orthotropic laminated plates subjected to shearing loads.

* Corresponding author. Tel.: +90 282 652 94 76; fax: +90 282 652 93 72.
E-mail addresses: buymaz@nku.edu.tr (B. Uymaz), metina@trakya.edu.tr (M. Aydogdu).
URLs: http://www.nku.edu.tr (B. Uymaz), http://www.trakya.edu.tr (M. Aydogdu).

0263-8223/$ - see front matter © 2012 Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.compstruct.2012.08.031

The critical buckling loads and the buckling mode shapes of plates which subjected to shear loads and axial loads are important characteristics in the design of plated structures and depend on boundary conditions, material characteristics and the plate geometry parameters such as aspect ratio and sid-to-thickness ratio [28,29].

In the present study, the shear buckling problem of FG plates is investigated by using the Ritz method based on the three-dimensional linear elasticity plate theory. Also, the results are compared with the results which obtained by using Ritz method based on the five-degree-of-freedom shear deformable plate theory and the Classical Plate Theory (CPT). Additionally, the shear buckling results compared with the axial buckling results which obtained considered theories. Three dimensional shear buckling solutions are obtained by using the Ritz method and assumed displacement functions are in the form of the triplicate series of Chebyshev polynomials multiplied by a boundary function which provide that the displacement components satisfies the geometric boundary conditions of the plate. Considered boundary conditions are simply supported (S) on all edges and clamped (C) on all edges of the plate. The effects of the material composition (p index), the aspect ratio (a/b) and the side-to-thickness ratio (a/h) on the critical shear buckling load of considered plate with different boundary conditions and different loading conditions are investigated. Three-dimensional deformed mode shapes are given by contour plots to enhance our understanding of the buckling characteristics of these plates.

2. Theoretical formulation

A rectangular plate subjected to uniformly distributed in-plane shear force $N^e_{xy} = -N_s$ is considered (Fig. 1). The perimeter of the plate is bounded by the edges, $-a/2 \leqslant x \leqslant a/2$ and $-b/2 \leqslant y \leqslant b/2$. The origin of the co-ordinate system (x, y, z) is placed at the geometric center of the plate and the axes are parallel to the edges of the plate and the corresponding displacement components U, V and W along the x, y and z directions, respectively.

The plate is made from a mixture of ceramic and metal, and material composition continuously varies through the thickness direction. The effective material properties are obtained by the power law distribution as follows,

$$P_e = V_m P_{em} + V_c P_{ec} \tag{1}$$

$$V_m + V_c = 1 \tag{2}$$

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{3}$$

$$P_e(z) = (P_{ec} - P_{em})\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_{em} \tag{4}$$

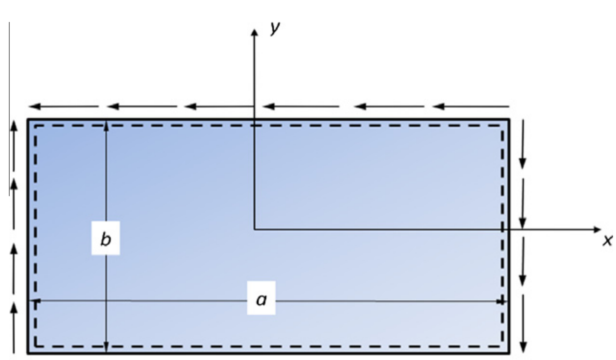

Fig. 1. Buckling of rectangular plates under the action of shearing stresses.

where P_e represents the effective material property which considered such as elasticity modulus E and the Poisson ratio ν; the subscripts c and m denote the ceramic and metal which corresponding the material property of the upper and lower surface of the plate, respectively. Where V_c is the volume fraction of the ceramic; and p is the volume fraction exponent, $0 \leqslant p \leqslant \infty$. When the volume fraction exponent is 0 plate is full of ceramic, and when the volume fraction exponent is 1 the variation of the volume fraction of the ceramic is linear. When the value of p increases hence V_c decreases, the content of metal considered plate increases.

2.1. Formulation based on three dimensional elasticity theory

From the three-dimensional elasticity theory the displacement field for the considered plate can be written as follows

$$U = U(x, y, z) \tag{5a}$$

$$V = V(x, y, z) \tag{5b}$$

$$W = W(x, y, z) \tag{5c}$$

A linear elastic material behavior is considered. The stress–strain relations are given by the generalized Hooke's law as follows

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ & C_{11} & C_{12} & 0 & 0 & 0 \\ & & C_{11} & 0 & 0 & 0 \\ & & & C_{66} & 0 & 0 \\ & \text{sym} & & & C_{66} & 0 \\ & & & & & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} \tag{6}$$

where C_{ij} are elastic constants and defined as follows

$$C_{11} = \frac{E(z)(1-\nu(z))}{(1+\nu(z))(1-2\nu(z))}, \quad C_{12} = \frac{\nu(z)E(z)}{(1+\nu(z))(1-2\nu(z))}$$
$$C_{66} = \frac{C_{11} - C_{12}}{2} \tag{7}$$

and ε_{ij} $(i, j = x, y, z)$ are the infinitesimal strain components as follows

$$\varepsilon_x = U_{,x}, \quad \varepsilon_y = V_{,y}, \quad \varepsilon_z = W_{,z} \tag{8a}$$

$$\gamma_{yz} = V_{,z} + W_{,y} \quad \gamma_{xz} = U_{,z} + W_{,x} \quad \gamma_{xy} = U_{,y} + V_{,x} \tag{8b}$$

where $(x, y, z) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$.

The total potential energy functional Π of the elastic plate is defined for a buckling problem as

$$\Pi = U_s + V_e \tag{9}$$

where U_s is the linear elastic strain energy, V_e is the work done by applied forces and given by

$$U_s = \frac{1}{2}\iiint_v (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \sigma_{xz}\varepsilon_{xz} + \sigma_{yz}\varepsilon_{yz} + \sigma_{xy}\varepsilon_{xy})dV \tag{10}$$

$$V_e = \frac{1}{2}\iiint_v \left(N^e_x(w_{,x})^2 + N^e_y(w_{,y})^2 + 2N^e_{xy}(w_{,x}w_{,y})\right)dV \tag{11}$$

where N^e_x, N^e_y are the in-plane axial forces and N^e_{xy} is the in-plane shear force, and $N^e_x < 0$ and $N^e_y < 0$ for compressive forces and $N^e_{xy} < 0$. As in the case of axial buckling the in-plane forces are defined as follows,

$$N^e_x = -N_0, \quad N^e_y = -\delta N_0, \quad \delta = \frac{N^e_y}{N^e_x} \text{ and } N^e_{xy} = 0 \tag{12}$$