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Static and dynamic analysis of laminated composite and sandwich plates and shells by using a new higher-order shear deformation theory

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ABSTRACT

A new higher order shear deformation theory for elastic composite/sandwich plates and shells is developed. The new displacement field depends on a parameter "m", whose value is determined so as to give results closest to the 3D elasticity bending solutions. The present theory accounts for an approximately parabolic distribution of the transverse shear strains through the shell thickness and tangential stress-free boundary conditions on the shell boundary surface. The governing equations and boundary conditions are derived by employing the principle of virtual work. These equations are solved using Navier-type, closed form solutions. Static and dynamic results are presented for cylindrical and spherical shells and plates for simply supported boundary conditions. Shells and plates are subjected to bi-sinusoidal, distributed and point loads. Results are provided for thick to thin as well as shallow and deep shells. The accuracy of the present code is verified by comparing it with various available results in the literature.

1. Introduction

Shells are curved structures, which have increased structural stiffness compared to plates in carrying loads and moments by a combined membrane and bending action due to their curvature. On the other hand, fiber reinforced plastic (FRP) composite materials provide high performance and reliability due to high strength-to-weight and high stiffness-to-weight ratios, excellent fatigue strength, resistance to corrosion (e.g. glass fiber composites), and most importantly the design flexibility also known as tailoring the materials for desired applications. As a result, shell structures made of composite materials will continue being widely used for many years in many engineering fields such as naval, aerospace, automotive and construction industries and for sporting goods, medical devices and many other areas [1–4].

It is important to mention that the study of shell theories allow the understanding of plates, curved beams, and flat beams as special cases. In the past four decades, a significant number of theories for composited laminated shells have been presented. These theories can be classified in different models, such as equivalent single layer, quasi-layerwise and layerwise models [5,6]. This paper considers an equivalent single layer model. Detailed information may be found for the rest of the models in the papers by Carrera [7–9] and Demasi [5,6,10–13]. According to Carrera's unified formulation (CUF) [14] or the unified generalized formulation (GUF) proposed by Demasi [5,6,11–13,15], among equivalent single layer models, there are many existing class of theories which CUF or GUF can be reproduced.

However, application of such theories to layered anisotropic composite shells can lead errors up to 30% in deflections, stresses and frequencies [69]. First order shear deformation theory (FSDT) is based on the kinematic field assumption postulated by Mindlin [16], in which the constant transverse shear strain components along the thickness is accounted for [17–19]. Higher order shear deformation theories (HSDTs) were developed to improve the analysis of shell responses and extensively used by many researchers [20–59]. Normally, these well-known theories comply with the free surface boundary conditions and account for approximately parabolic distribution of shear stresses through the thickness of the shell. More advanced HSDTs account for the continuity of the transverse shear stresses, and provide improved results [11–13].

Many higher order theories were proposed by Kant and Swaminathan [31], Reddy and Liu [24], Touratier [48], Soldatos [51]. The present HSDT is simple in the sense that it contains the same dependent unknowns as in the first order shear deformation theory. The present theory is based on a displacement field in which the displacements of the middle surface are expanded as a combination of exponential and polynomial functions of the thickness coordinate, and the transverse displacement is assumed to be constant through the shell thickness. The theory is constructed from 3D elasticity bending solutions of plates, by performing several computations of the present shell/plate governing equations, which has "m" parameter dependent.

The present theory accounts for approximate parabolic distribution of the transverse shear strains through the shell thickness and

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the tangential stress-free boundary conditions on the shell surface. The governing equations and boundary conditions are derived by employing the principle of virtual work. These equations are then solved via a Navier-type, closed form solution. Static and dynamic results are presented for cylindrical and spherical shells under simply supported boundary conditions. Shells and plates are subjected to bi-sinusoidal, distributed and localized loadings. Numerical results are generated for thick to thin as well as shallow and deep shells. The accuracy of the present code is ascertained by comparing it with various available results in the literature. It is found that the present theory gives more precise results than other higher order theories compared here.

2. Statement of the problem

The aim of this work is to establish a new shear deformation theory. The main idea of the present theory comes from the shear deformation theory lately proposed by Aydogdu [57]. If the shape strain functions of Karama et al. [55] and Aydogdu [57] are compared, it can be seen that mathematically both expressions are the same [55,57], see Eq. (7), and in author's opinion the way how the parameter "α" was obtained in [57], needs to be confirmed. In fact, the present shear deformation theory firstly clarifies the previous mentioned theory [57], and presents a new more precise theory for laminated composite sandwich plates and shell structures.

To formally address the shear deformation theory, assume with the following displacement field:

$$\begin{aligned}
\bar{u}(\xi_1,\xi_2,\xi_3;t) &= \left(1+\frac{\varsigma}{R_1}\right)u - \xi_3\frac{\partial w}{a_1\partial\xi_1} + f(\xi_3)\phi_1,\\
\bar{v}(\xi_1,\xi_2,\xi_3;t) &= \left(1+\frac{\varsigma}{R_2}\right)v - \xi_3\frac{\partial w}{a_2\partial\xi_2} + f(\xi_3)\phi_2,\\
\bar{w}(\xi_1,\xi_2;t) &= w.
\end{aligned} \tag{1a-c}$$

where $u(\xi_1,\xi_2;t)$, $v(\xi_1,\xi_2;t)$, $w(\xi_1,\xi_2;t)$, $\phi_1(\xi_1,\xi_2;t)$ and $\phi_2(\xi_1,\xi_2;t)$ are the five unknown displacement functions of the middle surface of the panel while "$f(\xi_3)$" represents shape functions determining the distribution of the transverse shear strains and stresses along the thickness.

Surveys of various shear deformation theories for plates (which can be adapted to shells) can be found in the works of Idibi et al. [52], Karama et al. [54], Aydogdu [57]. The shape functions developed by different researchers are presented chronologically as follow:

Ambartsumian [49],

$$f(\xi_3) = \frac{\xi_3}{2}\left[\frac{h^2}{4} - \frac{\xi_3^2}{3}\right], \tag{2}$$

Kaczkowski [66], Panc [67], Reissner [18],

$$f(\xi_3) = \frac{5\xi_3}{4}\left[1 - \frac{4\xi_3^2}{3h^2}\right], \tag{3}$$

Levinson [47], Murthy [68] and Reddy [23],

$$f(\xi_3) = \xi_3\left[1 - \frac{4\xi_3^2}{3h^2}\right], \tag{4}$$

Touratier [48],

$$f(\xi_3) = \frac{h}{\pi}\sin\left(\frac{\pi\xi_3}{h}\right), \tag{5}$$

Soldatos [51],

$$f(\xi_3) = h\sinh\left(\frac{\xi_3}{h}\right) - \xi_3\cosh\left(\frac{1}{2}\right), \tag{6}$$

Karama et al. [55] and Aydogdu [57],

$$f(\xi_3) = \xi_3 e^{-2(\xi_3/h)^2} = \xi_3\alpha^{-2(\xi_3/h)^2/\ln\alpha}, \quad \forall\alpha > 0, \tag{7}$$

and Mantari et al. [70],

$$f(\xi_3) = \sin\left(\frac{\pi\xi_3}{h}\right)e^{\frac{1}{2}\cos\left(\frac{\pi\xi_3}{h}\right)} + \frac{\pi}{2h}\xi_3. \tag{8}$$

In this paper, as abovementioned, a new shear deformation theory is developed. Following procedures similar to the ones presented by Reddy and Liu [24] or the generalized procedure developed by Soldatos [51] that satisfies tangential stress-free boundary conditions at the top and bottom surfaces of the panel, the new displacement field presented here is given as follows:

$$\begin{aligned}
\bar{u}(\xi_1,\xi_2,\xi_3;t) &= \left(1+\frac{\varsigma}{R_1}\right)u - \xi_3\frac{\partial w}{a_1\partial\xi_1} + \xi_3 m^{-2(\xi_3/h)^2}\phi_1,\\
\bar{v}(\xi_1,\xi_2,\xi_3;t) &= \left(1+\frac{\varsigma}{R_2}\right)v - \xi_3\frac{\partial w}{a_2\partial\xi_2} + \xi_3 m^{-2(\xi_3/h)^2}\phi_2,\\
\bar{w}(\xi_1,\xi_2;t) &= w.
\end{aligned} \tag{9a-c}$$

By comparing the Eqs. (9a-b) with (1a-b), $f(\xi_3)$ becomes,

$$f(\xi_3) = \xi_3 m^{-2(\xi_3/h)^2} + y\xi_3, \quad m \geqslant 0, \quad y = 0. \tag{10}$$

The function $f(\xi_3)$ in Eq. (10) is m dependent and therefore it needs to be calculated or selected. It is obtained after several computations of the shell or plate governing Eqs. (21a-e), to give the maximum center plate deflection which gives closest results compared to 3D elasticity bending solutions provided by Srinivas [62]. Calculation of the parameter will further be discussed after specifying the shell or plate governing equations.

The starting point of the present thick shell theory is the three-dimensional elasticity theory, expressed in general curvilinear (reference) surface-parallel coordinates; while the thickness coordinate is normal to the reference (middle) surface as given in Fig. 1. The strain–displacement relations, based on this formulation, are written as follows [71]:

$$\begin{aligned}
\varepsilon_1 &= \frac{1}{A_1}\left(\frac{\partial\bar{u}}{\partial\xi_1} + \frac{1}{a_2}\frac{\partial a_1}{\partial\xi_2}\bar{v} + \frac{a_1}{R_1}\bar{w}\right),\\
\varepsilon_2 &= \frac{1}{A_2}\left(\frac{\partial\bar{v}}{\partial\xi_2} + \frac{1}{a_1}\frac{\partial a_2}{\partial\xi_1}\bar{u} + \frac{a_2}{R_2}\bar{w}\right),\\
\varepsilon_4 &= \frac{1}{A_2}\frac{\partial\bar{w}}{\partial\xi_2} + A_2\frac{\partial}{\partial\xi_3}\left(\frac{\bar{v}}{A_2}\right),\\
\varepsilon_5 &= \frac{1}{A_1}\frac{\partial\bar{w}}{\partial\xi_1} + A_1\frac{\partial}{\partial\xi_3}\left(\frac{\bar{u}}{A_1}\right),\\
\varepsilon_6 &= \frac{A_2}{A_1}\frac{\partial}{\partial\xi_1}\left(\frac{\bar{v}}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial\xi_2}\left(\frac{\bar{u}}{A_1}\right),
\end{aligned} \tag{11a-f}$$

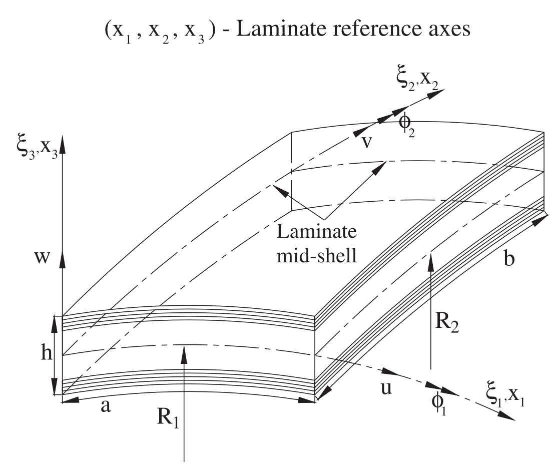

Fig. 1. Laminate geometry with positive set of lamina/laminate reference axes, displacement components and fiber orientation.