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Transient thermoelastic analysis for a multilayered hollow sphere with piecewise power law nonhomogeneity

Yoshihiro Ootao *

Department of Mechanical Engineering, Graduate School of Engineering, Osaka Prefecture University, 1-1 Gakuen-cho, Nakaku, Sakai 599-8531, Japan

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ABSTRACT

This paper is concerned with the theoretical treatment of transient thermoelastic problem involving a multilayered hollow sphere with piecewise power law nonhomogeneity due to uniform heat supply. The thermal and thermoelastic constants of each layer are expressed as power functions of the radial coordinate, and their values continue on the interfaces. We obtain the exact solution for the one-dimensional temperature change in a transient state, and thermoelastic response. Some numerical results for the temperature change, the displacement and the stress distributions are shown in figures.

1. Introduction

Functionally graded materials (FGMs) are nonhomogeneous material systems that two or more different material ingredients change continuously and gradually. In recent years, the concept of FGMs has been applied in many industrial fields in addition to the aerospace field [1,2]. When FGMs are used under high temperature conditions or are subjected to several thermal loading, it is necessary to analyze the thermal stress problems for FGMs. It is well-known that thermal stress distributions in a transient state can show large values compared with the one in a steady state. Therefore, the transient thermoelastic problems for FGMs become important. The governing equations for the temperature field and the associate thermoelastic field of FGMs become of nonlinear form in generally, the analytical treatment is very difficult. As the analytical treatment of the thermoelastic problems of FGMs, there are two pieces of treatment mainly. One is introducing the theory of laminated composites, which have a number of homogeneous layers along the thickness direction. Using the theory of laminated composites, we analyzed theoretically the transient thermal stress problems of several analytical models, i.e. hollow cylinders [3–6], plates [7,8] and hollow spheres [9,10]. Sugano et al. reported an approximate three-dimensional analysis of thermal stresses in a nonhomogeneous plate with temperature change and nonhomogeneous properties only in the thickness direction [11] and an one-dimensional analysis of transient thermal stress in a circular plate with arbitrary variation of heat-transfer coefficient [12].

The other analytical treatment is the exact analysis under the assumption that the material properties are given in the specific functions containing the variable of the thickness coordinate without using the laminated composite model. As the exact treatment without laminated composite models, Sugano analyzed exactly one-dimensional transient thermal stresses of nonhomogeneous plate where the thermal conductivity and Young's modulus vary exponentially, whereas Poisson's ratio and the coefficient of linear thermal expansion vary arbitrarily in the thickness direction [13]. Vel and Batra analyzed the three-dimensional transient thermal stresses of the functionally graded rectangular plate [14]. We analyzed the transient thermal stress problems of a functionally graded circular plate [15], a functionally graded thick strip [16] and a functionally graded rectangular plate [17], where the thermal conductivity, the coefficient of linear thermal expansion and Young's modulus vary exponentially in the thickness direction, due to nonuniform heat supply. On the other hand, examples of thermal buckling problems are as follows. Shariat and Eslami analyzed buckling of thick functionally graded plates under mechanical and thermal loads using the third order shear deformation theory [18]. Matsunaga analyzed thermal buckling of functionally graded plates using a 2D higher-order deformation theory [19]. Saidi and Baferani analyzed thermal buckling of moderately thick functionally graded annular sector plates using the first order shear deformation plate theory [20].

As exact analysis for shell type structures, the one-dimensional solutions for transient thermal stresses of functionally graded hollow cylinders and hollow spheres whose material properties vary with the power product form of radial coordinate variable were obtained [21,22]. Zhao et al. analyzed the one-dimensional

* Tel.: +81 72 254 9208; fax: +81 72 254 9904.
E-mail address: ootao@me.osakafu-u.ac.jp

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transient thermo-mechanical behavior of a functionally graded solid cylinder, whose thermoelastic constants vary exponentially through the thickness [23]. Shao et al. analyzed one-dimensional transient thermo-mechanical behavior of functionally graded hollow cylinders, whose thermoelastic constants are expressed as Taylor's series [24]. For the case of nonuniform distributed heating, Shao et al. obtained the analytical solutions for transient thermo-mechanical response of functionally graded cylindrical panels [25] and functionally graded hollow cylinders [26]. We obtained the two-dimensional analytical solution for transient thermal stresses of a functionally graded cylindrical panel whose material properties vary with the power product form of radial coordinate variable [27]. However, these studies discuss the thermoelastic problems of one-layered FGM models, which have the big limitation of nonhomogeneity. On the other hand, the arbitrary nonhomogeneity can be expressed in the theory of laminated composites approximately, but the material properties are discontinuous on the interfaces.

From the viewpoint of above mentioned, we analyze the transient thermoelastic analysis for a multilayered hollow sphere with piecewise power law nonhomogeneity as a new FGM model with arbitrary properties.

2. Analysis

Consider a multilayered hollow sphere with piecewise power law nonhomogeneity. The thermal and thermoelastic constants of each layer are expressed as power functions of the radial coordinate, and their values continue on the interfaces. The multilayered hollow sphere's inner and outer radii are defined r_a and r_b, respectively. Moreover, r_i is the outer radius of ith layer.

2.1. Heat conduction problem

It is assumed that the multilayered hollow sphere is initially at zero temperature and is suddenly heated from the inner and outer surfaces by surrounding media of constant temperatures T_a and T_b with the relative heat-transfer coefficients h_a and h_b. Then the temperature distribution shows a one-dimensional distribution, and the transient heat conduction equation for the ith layer is taken in the following forms:

$$c_i(r)\rho_i(r)\frac{\partial T_i}{\partial t} = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left[\lambda_i(r)r^2\frac{\partial T_i}{\partial r}\right] \tag{1}$$

The thermal conductivity λ_i and the heat capacity per unit volume $c_i\rho_i$ in each layer are assumed to take the following forms:

$$\lambda_i(r) = \lambda_i^0\left(\frac{r}{r_{i-1}}\right)^{m_i} \tag{2}$$

$$c_i(r)\rho_i(r) = c_i^0\rho_i^0\left(\frac{r}{r_{i-1}}\right)^{k_i} \tag{3}$$

where

$$m_i = \frac{\ln(\bar{\lambda}_{i+1}^0/\bar{\lambda}_i^0)}{\ln(\bar{r}_i/\bar{r}_{i-1})}, \quad k_i = \frac{\ln(\bar{c}_{i+1}^0\bar{\rho}_{i+1}^0/\bar{c}_i^0\bar{\rho}_i^0)}{\ln(\bar{r}_i/\bar{r}_{i-1})} \tag{4}$$

Substituting Eqs. (2) and (3) into Eq. (1), the transient heat conduction equations in dimensionless form are

$$\frac{\partial\bar{T}_i}{\partial\tau} = \bar{\kappa}_i^0\bar{r}_{i-1}^{k_i-m_i}\left[(m_i+2)\bar{r}^{m_i-k_i-1}\frac{\partial\bar{T}_i}{\partial\bar{r}} + \bar{r}^{m_i-k_i}\frac{\partial^2\bar{T}_i}{\partial\bar{r}^2}\right]; \quad i = 1,2,\ldots,N \tag{5}$$

The initial and thermal boundary conditions in dimensionless form are

$$\tau = 0; \quad \bar{T}_i = 0; \quad i = 1,2,\ldots,N \tag{6}$$

$$\bar{r} = \bar{r}_a; \quad \frac{\partial\bar{T}_1}{\partial\bar{r}} - H_a\bar{T}_1 = -H_a\bar{T}_a \tag{7}$$

$$\bar{r} = \bar{r}_i; \quad \bar{T}_i = \bar{T}_{i+1}; \quad i = 1,2,\ldots,N-1 \tag{8}$$

$$\bar{r} = \bar{r}_i; \quad \frac{\partial\bar{T}_i}{\partial\bar{r}} = \frac{\partial\bar{T}_{i+1}}{\partial\bar{r}}; \quad i = 1,2,\ldots,N-1 \tag{9}$$

$$\bar{r} = 1; \quad \frac{\partial\bar{T}_N}{\partial\bar{r}} + H_b\bar{T}_N = H_b\bar{T}_b \tag{10}$$

In Eqs. (4)–(10), we have introduced the following dimensionless values:

$$(\bar{T}_i,\bar{T}_a,\bar{T}_b) = \frac{(T_i,T_a,T_b)}{T_0}, \quad (\bar{r},\bar{r}_i,\bar{r}_a) = \frac{(r,r_i,r_a)}{r_b}, \quad \tau = \frac{\kappa_0 t}{r_b^2},$$
$$\kappa_0 = \frac{\lambda_0}{c_0\rho_0},$$
$$\bar{\kappa}_i^0 = \frac{\lambda_i^0}{c_i^0\rho_i^0}, \quad (\bar{\lambda}_i,\bar{\lambda}_i^0) = \frac{(\lambda_i,\lambda_i^0)}{\lambda_0}, \quad (\bar{c}_i\bar{\rho}_i,\bar{c}_i^0\bar{\rho}_i^0) = \frac{(c_i\rho_i,c_i^0\rho_i^0)}{c_0\rho_0},$$
$$(H_a,H_b) = (h_a,h_b)r_b \tag{11}$$

where T_i is the temperature change; t is time; and T_0, λ_0, $c_0\rho_0$ and κ_0 are typical values of temperature, thermal conductivity, heat capacity per unit volume and thermal diffusivity, respectively. Introducing the Laplace transformation with respect to the variable τ, the solution of Eq. (5) can be obtained so as to satisfy the conditions (6)–(10). This solution is shown as follows:

$$\bar{T}_i = \frac{1}{F}\left(\bar{A}_i'\bar{r}^{-m_i-1} + \bar{B}_i'\right) + \sum_{j=1}^{\infty}\frac{2}{\mu_{1j}\Delta'(\mu_{1j})}\bar{r}^{-(m_i+1)/2}$$
$$\times\exp\left[-\frac{\mu_{1j}^2}{4}\cdot\frac{\bar{\kappa}_1^0}{\bar{r}_a^{m_1-k_1}}(2-m_1+k_1)^2\tau\right]\times\left[\bar{A}_i J_{\gamma_i}\left(\Omega_i\mu_{1j}\bar{r}^{1-\frac{m_i-k_i}{2}}\right)\right.$$
$$\left.+\bar{B}_i Y_{\gamma_i}\left(\Omega_i\mu_{1j}\bar{r}^{1-\frac{m_i-k_i}{2}}\right)\right]; \quad i = 1,2,\ldots,N \tag{12}$$

where $J_x()$ and $Y_x()$ are the Bessel functions of the first and second kind of order x, respectively. And Δ and F are the determinants of $2N\times 2N$ matrix $[a_{kl}]$ and $[e_{kl}]$, respectively; the coefficients $\bar{A}_i$ and $\bar{B}_i$ are defined as the determinant of the matrix similar to the coefficient matrix $[a_{kl}]$, in which the $(2i-1)$th column or $2i$th column is replaced by the constant vector $\{c_k\}$, respectively. Similarly, the coefficients $\bar{A}_i'$ and $\bar{B}_i'$ are defined as the determinant of the matrix similar to the coefficient matrix $[e_{kl}]$, in which the $(2i-1)$th column or $2i$th column is replaced by the constant vector $\{c_k\}$, respectively. The nonzero elements of the coefficient matrices $[a_{kl}]$, $[e_{kl}]$ and the constant vector $\{c_k\}$ are given as

$$a_{1,1} = \bar{r}_a^{-\frac{m_1+1}{2}}\left\{\left[\left(1-\tfrac{m_1-k_1}{2}\right)\left(\gamma_1-\tfrac{m_1+1}{2-m_1+k_1}\right)\bar{r}_a^{-1}-H_a\right]J_{\gamma_1}\left(\mu_1\bar{r}_a^{1-\frac{m_1-k_1}{2}}\right)\right.$$
$$\left.-\mu_1\left(1-\tfrac{m_1-k_1}{2}\right)\bar{r}_a^{-\frac{m_1-k_1}{2}}J_{\gamma_1+1}\left(\mu_1\bar{r}_a^{-\frac{m_1-k_1}{2}}\right)\right\},$$
$$a_{1,2} = \bar{r}_a^{-\frac{m_1+1}{2}}\left\{\left[\left(1-\tfrac{m_1-k_1}{2}\right)\left(\gamma_1-\tfrac{m_1+1}{2-m_1+k_1}\right)\bar{r}_a^{-1}-H_a\right]Y_{\gamma_1}\left(\mu_1\bar{r}_a^{1-\frac{m_1-k_1}{2}}\right)\right.$$
$$\left.-\mu_1\left(1-\tfrac{m_1-k_1}{2}\right)\bar{r}_a^{-\frac{m_1-k_1}{2}}Y_{\gamma_1+1}\left(\mu_1\bar{r}_a^{-\frac{m_1-k_1}{2}}\right)\right\},$$
$$a_{2N,2N-1} = \left[\left(1-\tfrac{m_N-k_N}{2}\right)\left(\gamma_N-\tfrac{m_N+1}{2-m_N+k_N}\right)+H_b\right]J_{\gamma_N}(\mu_N)$$
$$-\mu_N\left(1-\tfrac{m_N-k_N}{2}\right)J_{\gamma_N+1}(\mu_N),$$
$$a_{2N,2N} = \left[\left(1-\tfrac{m_N-k_N}{2}\right)\left(\gamma_N-\tfrac{m_N+1}{2-m_N+k_N}\right)+H_b\right]Y_{\gamma_N}(\mu_N)$$
$$-\mu_N\left(1-\tfrac{m_N-k_N}{2}\right)Y_{\gamma_N+1}(\mu_N) \tag{13}$$