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A two-step optimization scheme for maximum stiffness design of laminated plates based on lamination parameters

Shutian Liu [a,*], Yupin Hou [a], Xiannian Sun [b], Yongcun Zhang [a]

[a] State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, China
[b] Transportation Equipment and Ocean Engineering College, Dalian Maritime University, Dalian 116026, China

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ABSTRACT

The purpose of this paper is to propose an effective solution scheme of simultaneous optimization design of layup configuration and fiber distribution for maximum stiffness design of laminated plates. Firstly, a numerical analysis of the lamination parameters feasible region for a laminated plate consisting of various given number of ply groups (each ply group may have different thickness and all the fibers in one ply group are orientated in an identical direction) is carried out, and it is found that the feasible region based on only a few ply groups is very close to the overall one determined by infinite plies. Therefore, it is suggested that the feasible region of lamination parameters of a laminated plate could be approximately determined by the layup configuration of least ply groups. Secondly, a two-step simultaneous optimization scheme of layup configuration and fiber distribution for maximum stiffness design of laminated plates is proposed. Accordingly, by using ply thickness, fiber orientation angle and fiber volume fraction in a laminated plate of least ply groups as design variables, the optimal lamination parameters for maximum stiffness is obtained. Then, taking the optimal lamination parameters as the design objective, a detailed layup design optimization is implemented by considering some limitations on manufacturing, such as preset ply thickness, and specific fiber orientation angle and a limited maximum number of consecutive plies in the same fiber orientation. Numerical examples are also presented to validate the proposed two-step optimization scheme.

1. Introduction

The mechanical properties of a structure are attributed to its material properties and the geometrical configuration (such as topology, shape, and size). Recently, a material and structure simultaneous optimization method was proposed to improve the structural properties [1–7]. The intention of this method is to simultaneously find an optimal material distribution in the global and local region within a structure, i.e., concurrently determine if a local region should be occupied by material or not and how to arrange achievable material to obtain the required material properties in the local region where material should be placed. To ensure that the designed material properties are realizable, it is necessary to identify the correlation among the material elastic properties, i.e., the feasible region of material elastic properties [8]. However, the feasible region of material properties is not easy to access and there is no general explicit formulation available for this feasible region so far. Nevertheless, it is much easier to define the feasible region of material properties for laminated composites because of its specific layup configuration which is determined by fiber volume fraction, thickness and fiber orientation angle of each ply. Therefore, it is a natural choice to practice the material and structure simultaneous optimization method on laminated composites.

A laminated plate consists of many plies and it is necessary to clarify some of the terminology that was included in this paper. A ply is referred to as a ply which is a flat (sometimes curved in a shell) arrangement of unidirectional fibers or woven fibers in a matrix. A laminate is a bonded stack of plies with various fiber orientations and all the plies have the same thickness. A ply group may have an arbitrary thickness, forming by one or more consecutive plies, and all the fibers in one ply group are orientated in an identical direction, which can be treated as a thicker ply in analysis.

The mechanical properties of a laminate are determined by its configuration, including fiber volume fraction and in-plane fiber distribution of each single ply and layup arrangement (such as the number of plies, and ply thickness and fiber orientation angle) [9,10]. Thus, it is possible for fiber reinforced composite structures to acquire the required structural properties by altering the configuration of laminated plates except for changing its geometrical

* Corresponding author. Tel./fax: +86 411 84706149.
E-mail address: stliu@dlut.edu.cn (S. Liu).

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size. Therefore, the mechanical potential of composite laminates can be maximized by an effective optimization design of in-plane fiber distribution and layup arrangement for a given fiber volume fraction. In this paper, the material and structure simultaneous optimization method [11,12] was employed to establish simultaneous optimization scheme of layup configuration and in-plane fiber distribution for maximum stiffness design of laminated plates.

In the conventional design, the material properties of a single ply is always assigned first and the laminate plates are directly designed by adjusting ply thickness and fiber orientation angle [13–15]. If the number of plies is not prescribed, one needs to solve an optimization problem involving a discrete variable (the number of plies). If the number of plies is large, there are too many design variables (including ply thickness and fiber orientation angle) to solve the corresponding optimization problem. On the other hand, the mechanical properties of laminated plates are related to some multiple-valued trigonometric functions of each ply orientation angle. Therefore, it is very difficult to design laminated plates directly by ply thickness and fiber orientation angle due to the existence of discrete variable and multiple-valued trigonometric functions.

For fiber reinforced composite laminates with the same material properties of each ply, the stiffness properties can be denoted by 12 lamination parameters which represent the layup configuration of laminates (fiber orientation angles, ply thickness and number of plies) [16]. According to the classic plate theory and Mindlin plate theory, these 12 lamination parameters correspond to its in-plane, bending and shear stiffness. It has been shown that the relationship between the stiffness properties and lamination parameters of laminated plates is convex and it is easier to design laminated plates by finding the optimal lamination parameters [17–20]. Therefore, optimal design of laminated plates based on lamination parameters is drawing more and more attentions due to its unique convex quality [21–25]. However, the stiffness properties of laminated plates are not completely independable to each other which implies that there are some correlations between 12 lamination parameters, i.e., there exists a feasible region for 12 lamination parameters to ensure that all the 12 lamination parameters can be simultaneously realized by one layup configuration. Consequently, the optimal lamination parameters should be searched within the corresponding feasible region. As a result, the key to the optimal design based on lamination parameters is to determine the feasible region of lamination parameters. The feasible region of 4 lamination parameters alone, corresponding to the in-plane and out-of-plane stiffness, respectively, was analytically given in [26–28]. However, an explicit formulation of the feasible region for all the 12 lamination parameters is still unavailable. An approach to solve an approximate feasible region was presented by [29–32] and the corresponding expressions of the approximate feasible region were provided. In the meantime, [31] also indicated that these expressions are necessary but not sufficient conditions for the feasible region of lamination parameters. In other words, the lamination parameters satisfying these expressions may not be able to be realized by any layup configuration. Therefore, it is necessary to further study the feasible region of lamination parameters.

Lamination parameters are the functions of ply thickness and fiber orientation angle of each ply and therefore lamination parameters determined by the realizable ply thicknesses and orientation angles must belong to the feasible region of lamination parameters. As mentioned above, it is very difficult to conduct an optimization of laminated plates directly by using ply thicknesses and fiber orientation angles as design variables when the number of plies is uncertain or large. However, a numerical analysis of the lamination parameters feasible region in this paper showed that the feasible region based on only a few thicker plies, i.e., ply groups was very close to the overall one determined by infinite plies. Therefore, it was suggested the feasible region of lamination parameters of a laminate could be approximately determined by the layup configuration of least ply groups.

Meanwhile, there are many limitations on ply thicknesses and fiber orientation angles due to the manufacturing or design requirements, which should be also taken into account in an optimization of laminated plates, such as the minimum thickness of a ply, specific fiber orientation angle (for example, 0°, ±45°, 90° only), and the maximum number of consecutive plies in the same fiber orientation, [33].

In this paper, firstly, a numerical analysis of the lamination parameters feasible region for various given number of plies was carried out, and the least ply groups (6 ply groups was chosen in this paper) was proposed to approximately determine the overall lamination parameters feasible region. Secondly, a two-step simultaneous optimization scheme of layup configuration and in-plane fiber distribution for maximum stiffness design of laminated plates was proposed. The thickness and fiber orientation angle and fiber volume fraction of least ply groups were taken as design variables, then the optimal lamination parameters for maximum stiffness was obtained. By taking the optimal lamination parameters as the design objective, a detailed layup design optimization was implemented by considering some limitations on manufacturing, such as the preset ply thickness, and specific fiber orientation angle and a given maximum number of consecutive plies in the same fiber orientation. Numerical examples were presented to validate the proposed two-step optimization scheme.

2. Problem formulation

2.1. Constitutive law and stiffness matrix for fiber reinforced laminates

Based on Mindlin plate theory, the constitution equation for a laminated plate can be written as

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{V} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{B} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^0 \\ \boldsymbol{\kappa} \\ \boldsymbol{\gamma}^0 \end{Bmatrix} \tag{1}$$

where $\mathbf{N}$, $\mathbf{M}$ and $\mathbf{V}$ are the stress, moment, and transverse shear stress resultants, respectively; $\boldsymbol{\varepsilon}^0$, $\boldsymbol{\kappa}$ and γ^0 are the strains, the curvatures at the mid-plane, and the transverse shear strains, respectively; and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$, $\mathbf{H}$ are the in-plane, coupling, out-of-plane, and shear stiffness, respectively. The stiffness components A_{ij}, B_{ij}, D_{ij} (i, j = 1, 2, 6), H_{ij} (i, j = 1, 2), can be expressed as follows:

$$A_{ij} = h\mathbf{K}, \quad B_{ij} = \frac{h^2}{4}\mathbf{K}, \quad D_{ij} = \frac{h^3}{12}\mathbf{K}, \quad (i,j = 1,2,6) \tag{2}$$

where h is the thickness of laminated plates. Matrix $\mathbf{K}$ is defined as

$$\mathbf{K} = \begin{bmatrix} 1 & \xi_1^R & \xi_3^R & 0 & 0 \\ 1 & -\xi_1^R & \xi_3^R & 0 & 0 \\ 0 & 0 & -\xi_3^R & 1 & 0 \\ 0 & 0 & -\xi_3^R & 0 & 1 \\ 0 & \xi_2^R/2 & \xi_4^R & 0 & 0 \\ 0 & \xi_2^R/2 & -\xi_4^R & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix}, \quad R = (A, B, D) \tag{3}$$

and H_{ij} (i, j = 1, 2) can be written as

$$\begin{Bmatrix} H_{11} \\ H_{22} \\ H_{12} \end{Bmatrix} = \frac{5h}{6} \begin{bmatrix} 1 & \xi_1^A \\ 1 & -\xi_1^A \\ 0 & -\xi_2^A \end{bmatrix} \begin{Bmatrix} U_6 \\ U_7 \end{Bmatrix} \tag{4}$$

where ξ_i^R $(i = 1, \ldots 4; R = A, B, D)$ in Eqs. (3) and (4) are lamination parameters, which are defined as