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Nonlinear theories of axisymmetric bending of functionally graded circular plates with modified couple stress

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ABSTRACT

A microstructure-dependent nonlinear theory for axisymmetric bending of circular plates, which accounts for through-thickness power-law variation of a two-constituent material, is developed using the principle of virtual displacements. The formulation is based on a modified couple stress theory, power-law variation of the material, temperature-dependent properties, and the von Kármán geometric nonlinearity. Classical and first-order shear deformation theories are considered in the study. The modified couple stress theory contains a material length scale parameter that can capture the size effect in a functionally graded material plate. The theories presented herein can be used to develop analytical solutions of bending, buckling, and free vibration for the linear case and finite-element models for the nonlinear case to determine the effect of the geometric nonlinearity, power-law index, and microstructure-dependent constitutive relations on linear and nonlinear response of axisymmetric analysis of circular plates.

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1. Background

1.1. Microstructural effects

In recent years a number of attempts have been made to bring microstructural length scales into the continuum description of beams and plates. Microstructure-dependent theories were developed for the Bernoulli–Euler beam by Park and Gao [1,2] and for the Timoshenko and Reddy-Levinson beams and Mindlin plates by Ma et al. [3–5] using a modified couple stress theory proposed by Yang et al. [6], which contains only one material length scale parameter. Recently, a complete formulation of functionally graded beams and plates with microstructure-dependent constitutive relations for Bernoulli–Euler and Timoshenko beams and third-order plate theories were developed by Reddy [7], Reddy and Kim [8], and Reddy and Srinivasa [9]. The formulation accounted for temperature-dependent material properties and the von Kármán nonlinearity, which may have significant contribution to the response of beam-like and plate-like elements used in micro- and nano-scale devices such as biosensors and atomic force microscopes (see, for example, Li et al. [10], and Pei et al. [11]). The present paper is an extension of the work in [7] to axisymmetric circular plates. The following review of literature provides a background for the present study.

1.2. Functionally graded materials

Functionally graded materials (FGMs) are those in which the volume fractions of the two or more materials are varied continuously as a function of position along certain dimension(s) of the structure [12]. These materials are mainly designed for high temperature environments, such as in aerospace, turbine rotors, flywheels and gears, just to mention a few. As the use of FGMs increases, new methodologies need to be developed to manufacture, characterize, analyze, and design structural components made of these materials. Two-constituent FGMs are usually made of a mixture of ceramic and metals for use in thermal environments. The ceramic constituent of the material provides the high temperature resistance due to its low thermal conductivity. The ductile metal constituent, on the other hand, prevents fracture due to high temperature gradient in a very short period of time.

A number of investigations dealing with FGM beams and plates have appeared [12–20]. In [16,18–20], the von Kármán nonlinearity was considered. To the best of our knowledge, no work has been reported till date which concerns the thermomechanical analysis of FGM circular plates with temperature-dependent properties, microstructural features, and the von Kármán nonlinearity. The present study is an attempt to fill this gap in the literature.

1.3. Present study

The objective of the current paper is to develop classical and first-order plate theories for axisymmetric bending of circular

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plates, accounting for through-thickness power-law variation of a two-constituent material with temperature dependent material properties, modified couple stress theory, and the von Kármán nonlinear strains. In particular, we extend the modified couple stress theory of Yang et al. [6] to the case of functionally graded axisymmetric circular plates with the von Kármán nonlinearity, and derive the equations of motion using Hamilton's principle. Since nanoscale devices may involve circular plate elements that may be functionally graded and undergo moderately large rotations, the newly developed theories can be used to capture the size effects in functionally graded micro circular plates. Moreover, the bending-extensional coupling is captured through the von Kármán nonlinear strains. The governing equations of the classical and first-order shear deformation plate theories with the microstructure-dependent length scale parameter are developed systematically using Hamilton's principle.

2. Constitutive models

2.1. Material variation through the thickness

Consider a solid circular plate of radius a and total thickness h. The r-coordinate is taken radially outward from the center the plate, z-coordinate along the thickness (or height) of the plate, and the θ-coordinate is taken along a circumference of the plate. In a general case where applied loads and geometric boundary conditions are not axisymmetric, the displacements (u_r, u_θ, u_z) along the coordinates (r, θ, z) are functions of r, θ, and z coordinates and time t. We assume that the material of the plate is isotropic but varies from one kind of material at the bottom, $z = -h/2$, to another material on the top, $z = h/2$. A typical material property of the FGM through the plate thickness is assumed to be represented by a power-law (see [16,18])

$$P(z,T) = [P_c(T) - P_m(T)]f(z) + P_m(T), \quad f(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{2.1}$$

where P_c and P_m are the values of a typical material property, such as the modulus, density, and conductivity, of a ceramic material and metal, respectively; n denotes the volume fraction exponent, called power-law index. The Poisson's ratio will be assumed to be constant throughout. When $n = 0$, we obtain the single-material plate (with property P_c).

When FGMs are used in high-temperature environment, the material properties are temperature-dependent and they can be expressed as (see [7])

$$P_\alpha(T) = c_0\left(c_{-1}T^{-1} + 1 + c_1T + c_2T^2 + c_3T^3\right), \quad \alpha = c \text{ or } m \tag{2.2}$$

where c_0 is a constant appearing in the cubic fit of the material property with temperature; and c_{-1}, c_1, c_2, and c_3 coefficients of T^{-1}, T, T^2, and T^3, obtained after factoring out c_0 from the cubic curve fit of the property. The modulus of elasticity, conductivity, and the coefficient of thermal expansion are considered to vary according to Eqs. (2.1) and (2.2).

2.2. Modified couple stress theory

According to the modified couple stress theory, the strain energy of an elastic beam can be expressed as

$$U = \frac{1}{2}\int_0^a \left[\int_{-\frac{h}{2}}^{\frac{h}{2}} (\boldsymbol{\sigma} : \boldsymbol{\varepsilon} + \mathbf{m} : \boldsymbol{\chi})dz\right] rdr \tag{2.3}$$

where a is radius of the plate, $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{m}$ is the deviatoric part of the symmetric couple stress tensor, $\boldsymbol{\varepsilon}$ is the von Kármán strain tensor, and $\boldsymbol{\chi}$ is the symmetric curvature tensor

$$\boldsymbol{\chi} = \frac{1}{2}\left[\nabla\boldsymbol{\omega} + (\nabla\boldsymbol{\omega})^{\mathrm{T}}\right] \tag{2.4}$$

where $\boldsymbol{\omega}$ is the rotation vector

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \mathbf{u} \tag{2.5}$$

The components of the von Kármán strain tensor in cylindrical coordinate system for the axisymmetric case are given by

$$\begin{aligned} \varepsilon_{rr} &= \frac{\partial u_r}{\partial r} + \frac{1}{2}\left(\frac{\partial u_z}{\partial r}\right)^2 \\ \varepsilon_{rz} &= \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} + \frac{\partial u_z}{\partial r}\frac{\partial u_z}{\partial z}\right) \\ \varepsilon_{\theta\theta} &= \frac{u_r}{r}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z} \end{aligned} \tag{2.6}$$

where (u_r, u_z) denote the total displacements along the (r,z) coordinates $(u_\theta = 0)$. The only nonzero component of the rotation vector is

$$\omega_\theta = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) \tag{2.7}$$

Hence, the nonzero components of the curvature tensor are

$$\chi_{r\theta} = \frac{1}{2}\left(\frac{\partial \omega_\theta}{\partial r} - \frac{\omega_\theta}{r}\right), \quad \chi_{z\theta} = \frac{1}{2}\frac{\partial \omega_\theta}{\partial z} \tag{2.8}$$

3. Classical plate theory

3.1. Displacements and strains

The total displacements (u_r, u_θ, u_z) along the three coordinate directions (r, θ, z), as implied by the Love–Kirchhoff hypothesis for plates, which is the same as the Euler–Bernoulli hypothesis for beams, are assumed in the form

$$u_r(r,z,t) = u(r,t) - z\frac{\partial w}{\partial r}, \quad u_\theta(r,z,t) = 0, \quad u_z(r,z,t) = w(r,t) \tag{3.1}$$

where u is the radial displacement and w is the transverse deflection of the point $(r,0)$ on the midplane of the plate at time t. The displacement field (3.1) is based on the Kirchhoff hypothesis that straight lines normal to the midplane before deformation remain (i) inextensible, (ii) straight, and (iii) normal to the midsurface after deformation. The Kirchhoff hypothesis amount to neglecting both transverse shear and transverse normal effects, that is, deformation is due entirely to bending and inplane stretching.

The von Kármán strains (2.6) for the classical plate theory take the form

$$\varepsilon_{rr} = \varepsilon_{rr}^{(0)} + z\varepsilon_{rr}^{(1)}, \quad \varepsilon_{\theta\theta} = \varepsilon_{\theta\theta}^{(0)} + z\varepsilon_{\theta\theta}^{(1)} \tag{3.2}$$

where

$$\begin{aligned} \varepsilon_{rr}^{(0)} &= \frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2, \quad \varepsilon_{rr}^{(1)} = -\frac{\partial^2 w}{\partial r^2} \\ \varepsilon_{\theta\theta}^{(0)} &= \frac{u}{r}, \quad \varepsilon_{\theta\theta}^{(1)} = -\frac{1}{r}\partial w \partial r \end{aligned} \tag{3.3}$$

The curvature component is

$$\begin{aligned} \omega_\theta &= \frac{1}{2}\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) = -\frac{\partial w}{\partial r}, \\ \chi_{r\theta} &= \frac{1}{2}\left(\frac{\partial \omega_\theta}{\partial r} - \frac{\omega_\theta}{r}\right) = \frac{1}{2}\left(-\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \end{aligned} \tag{3.4}$$

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