



Review

A new FE model based on higher order zigzag theory for the analysis of laminated sandwich beam with soft core

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ABSTRACT

A new finite element (FE) model has been developed based on higher order zigzag theory (HOZT) for the static analysis of laminated sandwich beam with soft core. In this theory, the in-plane displacement variation is considered to be cubic for both the face sheets and the core. The transverse displacement is assumed to vary quadratically within the core while it remains constant in the faces beyond the core. The proposed model satisfies the condition of transverse shear stress continuity at the layer interfaces and the zero transverse shear stress condition at the top and bottom of the beam. The nodal field variables are chosen in an efficient manner to overcome the problem of continuity requirement of the derivatives of transverse displacements. A C_0 quadratic beam finite element is implemented to model the HOZT for the present analysis. Numerical examples covering different features of laminated composite and sandwich beams are presented to illustrate the accuracy of the present model. Many new results are also presented which should be useful for future research.

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1. Introduction

The need for lighter, stronger and stiffer structure has motivated the use of composite laminates and sandwich structures. Sandwich construction is a special type of laminated structure with low strength core and high strength face sheets in the form of com-

posite laminates. Laminated composite (e.g., GFRP, CFRP etc.) structures are weak in shear due to their low shear modulus compared to extensional rigidity. Thus the effect of shear deformation is quite significant and it becomes more complex in case of sandwich construction, as the material property variation is very large between the core and face layers.

Considering this aspect in view a number of theories have been proposed for accurate analysis of composite laminates. Based on their assumed displacement fields, theories can be grouped as: (1) single layer theory, (2) layer-wise theory and (3) zigzag theory.

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In Single layer theory, the deformation of plate is expressed in terms of unknown parameters of the reference plane. In this theory the transverse shear strain is assumed to be uniform over the entire plate thickness (i.e., the transverse shear stress is constant) and it is known as Reissner–Mindlin's plate theory which is also called as first order shear deformation theory (FSDT). Goyal and Kapania [1] developed a five node beam FE model based on FSDT. However, this theory (FSDT) requires a shear correction factor to compensate for the actual parabolic variation of the shear stress.

A further improvement in this direction came in the form of higher order shear deformation theories (HSDT). In this theory the actual warping of the cross section is considered by taking higher order variation of in-plane displacement and as such there is no need of any shear correction factor [2]. Murthy et al. [3] presented a FE model based on third order shear deformation theory (i.e., HSDT) for analysis of asymmetrically stacked laminated beams. The beams were analyzed under uniformly distributed load for different boundary conditions. Based on HSDT, Subramanian [4] had presented a two node C_1 continuous FE beam model with eight degrees of freedom per node for flexural analysis of symmetrical laminated composite beams. The in-plane displacement and the out of plane displacement were assumed to be quadratic and quartic respectively. The displacement fields also satisfied the stress free conditions at top and bottom surfaces of beam. To avoid layer dependent shear correction factor, a parabolic variation of shear stress was assumed across the thickness of the beam. HSDT gives a continuous variation of the transverse shear strain across the thickness but shows discontinuity in the shear stress distribution at the layer interfaces due to different values of shear rigidity at the adjacent layers. But the actual behavior of a composite laminate is opposite i.e., the transverse shear stress must be continuous at the layer interface and the corresponding strain may be discontinuous [5].

In order to overcome the above disparity, the layer-wise theories are developed. In this theory unknown displacement components are taken at all the layer interfaces. Discrete layer theories proposed by Robbins and Reddy [6], Toledano and Murakami [7], Lu and Liu [8] and many others assume unique displacement field in each layer and displacement continuity across the layers. The performance of this plate theory is good but it required huge computational involvement as the number of unknowns increases directly with the increase in the number of layers.

The above problem has been subsequently overcome by defining the unknowns at different interfaces in terms of those at the reference plane. These theories are known as zigzag theories in general. In this theory, the in plane displacements have piece-wise variation across the plate thickness and the number of unknowns are made independent of the number of layers by equating the transverse shear stresses at the layer interfaces of the laminate. In some improved version of these theories, the condition of zero transverse shear stresses at the plate/beam top and bottom was also satisfied. The theories developed by Murakami [9], Di Sciuva [10], Lee et al. [11], Cho and Parmeter [12], Cho and Averill [13] and many other fall under this category. Zigzag models for laminated composite beams were also developed by using trigonometric terms to represent the linear displacement field and transverse shear strains and stresses [14,15]. Icardi [16] developed a single layer model for the analysis of laminated composite beam which appeared accurate only when the material properties were not changing abruptly across the thickness. So he presented four new sub-laminate models [17] for the analysis of sandwich beam. Four combinations i.e. linear or cubic approximation, with and without zigzag representation of displacement within the sub-laminates were considered in the model. Recently, Tahani [18] presented the analytical solution for laminated beams by using two theories based on layer wise displacement fields. These theories (Zigzag)

provide a very accurate approximation of the structural behavior even for lower span to thickness ratio. However, the zigzag theory has a problem in its finite element implementation as it requires C_1 continuity of the transverse displacement at the nodes.

Yip and Averill [19] developed sub-laminate models for laminated beams and plates which combined the benefits of the discrete layer wise and higher order zigzag theories. In this theory (i.e., sub-laminate theory), if multiple sub-laminates are needed to model a laminate, then additional shear stress degrees of freedom are also required which makes the element unsuitable for implementation into commercial FE codes. Cho and Averill [20] presented an improved sub-laminate model with first order zigzag approximation of displacement within each sub-laminate and developed an eight node C_0 FE having five displacement degrees of freedom in each node for each sub-laminate.

Averill [21] developed a C_0 finite element based on first order zigzag theory and overcome the C_1 continuity requirement by incorporating the concepts of independent interpolations and penalty functions. Hermitian functions were used by Di Sciuva [22,23] to approximate the transverse displacement in his formulations. Carrera [24] used two different fields along the laminate thickness direction for displacement and transverse shear stress respectively for his formulation. Averill and Yip [25] developed a C_0 finite element based on cubic zigzag theory, using interdependent interpolations for transverse displacement and rotations and penalty function concepts.

Aitharaju and Averill [26] developed a new C_0 FE based on a quadratic zigzag layer-wise theory. For eliminating shear locking phenomenon, the shear strain field is also made field consistent. The transverse normal stress was assumed to be constant through the thickness of the laminate. The new FE was applied to model the beam as combination of different sub-laminates.

Bambole and Desai [27] presented a nine node two dimensional hybrid interface FE based on minimum potential energy for the analysis of composite beam. The transverse stress components were treated as degrees of freedom at interface nodes with some displacement degrees of freedom which has given rise to a total of 24 degrees of freedom for the element. Results were also presented for sandwich beam with clamped boundary conditions.

Use of relatively soft core has been successfully implemented in many recent engineering applications for resisting the effect of blast or impact loading in structures. However, in case of soft core the transverse deformation may be significant and it is necessary to control the deflection within permissible limit. As such the variation of transverse deflection across the depth of a sandwich structure having soft core has to be calculated with sufficient accuracy. Frosting [28] has presented the classical and the higher order computational models of unidirectional sandwich panels with incompressible and compressible cores to demonstrate the differences in overall response of the panels as well as in the vicinity of the localized loads and supports.

To represent the variation of transverse deflection in a laminated sandwich structure it is required to introduce unknown transverse displacement fields across the depth in addition to that in the reference plane. This can be done by using sub-laminate plate theories but the number of unknowns will increase with the increase in the number of sub-laminates. On the other hand, introduction of additional unknowns in the transverse displacement fields invites additional C_1 continuity requirements in its finite element implementation by using the zigzag theory as mentioned earlier. Moreover, the application of a C_1 continuous finite element is not encouraged in a practical analysis.

Pandit et al [29] proposed a higher order zigzag theory for the analysis of sandwich plates with soft compressible core. To overcome the above problem of C_1 continuity they have used separate shape functions to define the derivatives of transverse

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