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A meshless local radial point collocation method for free vibration analysis of laminated composite plates

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ABSTRACT

In this paper, a meshless local radial point collocation method based on multiquadric radial basis function is proposed to analyze the free vibration of laminated composite plates. This method approximates the governing equations based on first-order shear deformation theory using the nodes in the support domain of any data center. Natural frequencies of the laminated composite plates with various boundary conditions, side-to-thickness ratios, and material properties are computed by present method. The choice of shape parameter, effect of dimensionless sizes of the support domain on accuracy, convergence characteristics are studied by several numerical examples. The results are compared with available published results which demonstrate the accuracy and efficiency of present method.

1. Introduction

Numerous scholars have investigated the meshless methods in recent years. Meshless methods can be categorized into three classes: meshless methods based on weak-forms, meshless methods based on collocation techniques, meshless methods based on the combination of weak-forms and collocation techniques [1]. Meshless methods based on collocation techniques include meshless global collocation method and meshless local collocation method. The meshless global collocation method approximates the solution of partial differential equations using all nodes in the problem domain. Ferreira et al. [2–9] and Xiang et al. [10–12] had used it to analyze the free vibration and static deformation of laminated composite plates. But global collocation method can result in fully populated coefficient matrices. To circumvent these difficulties a local collocation method which approximates the solution of partial differential equations using the nodes in the support domain of any data center has been proposed by Liu et al. [13–15], Lee et al. [16]. In Liu et al. [13], a local RBF collocation approach using a Hermite-type interpolation scheme was proposed to study the Cook's membrane problem. In the study of Liu et al. [14], a local radial point interpolation collocation method (RPICM) was presented to solve partial differential equations. The choice of shape parameter, the enforcement of additional polynomial terms, and the application of the Hermite-type interpolation were studied by several numerical examples. Liu et al. [15] proposed a stabilized local radial point collocation method (RPCM) based on least-squares stabilization technique to perform adaptive analysis. Lee et al. [16] presented the local multiquadric and the local inverse multiquadric approximations to solve boundary value problems. A local radial basis function based gridfree scheme has also been developed to solve unsteady, incompressible Navier–Stokes equations in primitive variables by Sanyasiraju and Chandhini [17]. An improved localized radial basis function meshless method was developed for computational aeroacoustics by Li et al. [18].

This paper deals with the free vibration analysis of laminated composite plates by a meshless local radial point collocation method based on multiquadric radial basis function. The choice of shape parameter, effect of dimensionless sizes of the support domain on accuracy, convergence characteristics are studied by several numerical examples. The present results are compared with the previous literatures. The aim of the present paper is to explore the potential of meshless local radial point collocation method in the free vibration analysis of laminated composite plates.

2. Governing equations based on first-order shear deformation theory

Governing equations based on first-order shear deformation theory are as follows [20]:

$$A_{11}\frac{\partial^2 u}{\partial x^2}+2A_{16}\frac{\partial^2 u}{\partial x\partial y}+A_{66}\frac{\partial^2 u}{\partial y^2}+A_{16}\frac{\partial^2 v}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y}+A_{26}\frac{\partial^2 v}{\partial y^2}+B_{11}\frac{\partial^2 \phi_x}{\partial x^2}+2B_{16}\frac{\partial^2 \phi_x}{\partial x\partial y}+B_{66}\frac{\partial^2 \phi_x}{\partial y^2}+B_{16}\frac{\partial^2 \phi_y}{\partial x^2}+(B_{12}+B_{66})\frac{\partial^2 \phi_y}{\partial x\partial y}+B_{26}\frac{\partial^2 \phi_y}{\partial y^2}=-I_0\omega^2 u-I_1\omega^2\phi_x \quad (1)$$

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$$A_{16}\frac{\partial^2 u}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y}+A_{26}\frac{\partial^2 u}{\partial y^2}+A_{66}\frac{\partial^2 v}{\partial x^2}+2A_{26}\frac{\partial^2 v}{\partial x\partial y}+A_{22}\frac{\partial^2 v}{\partial y^2}+B_{16}\frac{\partial^2 \phi_x}{\partial x^2}+(B_{12}+B_{66})\frac{\partial^2 \phi_x}{\partial x\partial y}+B_{26}\frac{\partial^2 \phi_x}{\partial y^2}+B_{66}\frac{\partial^2 \phi_y}{\partial x^2}+2B_{26}\frac{\partial^2 \phi_y}{\partial x\partial y}+B_{22}\frac{\partial^2 \phi_y}{\partial y^2}=-I_0\omega^2 v-I_1\omega^2\phi_y \tag{2}$$

$$A_{55}\frac{\partial^2 w}{\partial x^2}+2A_{45}\frac{\partial^2 w}{\partial x\partial y}+A_{44}\frac{\partial^2 w}{\partial y^2}+A_{55}\frac{\partial \phi_x}{\partial x}+A_{45}\frac{\partial \phi_x}{\partial y}+A_{45}\frac{\partial \phi_y}{\partial x}+A_{44}\frac{\partial \phi_y}{\partial y}=-I_0\omega^2 w \tag{3}$$

$$B_{11}\frac{\partial^2 u}{\partial x^2}+2B_{16}\frac{\partial^2 u}{\partial x\partial y}+B_{66}\frac{\partial^2 u}{\partial y^2}+B_{16}\frac{\partial^2 v}{\partial x^2}+(B_{12}+B_{66})\frac{\partial^2 v}{\partial x\partial y}+B_{26}\frac{\partial^2 v}{\partial y^2}-A_{45}\frac{\partial w}{\partial y}-A_{55}\frac{\partial w}{\partial x}+D_{11}\frac{\partial^2 \phi_x}{\partial x^2}+2D_{16}\frac{\partial^2 \phi_x}{\partial x\partial y}+D_{66}\frac{\partial^2 \phi_x}{\partial y^2}-A_{55}\phi_x+D_{16}\frac{\partial^2 \phi_y}{\partial x^2}+(D_{12}+D_{66})\frac{\partial^2 \phi_y}{\partial x\partial y}+D_{26}\frac{\partial^2 \phi_y}{\partial y^2}-A_{45}\phi_y=-I_1\omega^2 u-I_2\omega^2\phi_x \tag{4}$$

$$B_{16}\frac{\partial^2 u}{\partial x^2}+(B_{12}+B_{66})\frac{\partial^2 u}{\partial x\partial y}+B_{26}\frac{\partial^2 u}{\partial y^2}+B_{66}\frac{\partial^2 v}{\partial x^2}+2B_{26}\frac{\partial^2 v}{\partial x\partial y}+B_{22}\frac{\partial^2 v}{\partial y^2}-A_{44}\frac{\partial w}{\partial y}-A_{45}\frac{\partial w}{\partial x}+D_{16}\frac{\partial^2 \phi_x}{\partial x^2}+D_{26}\frac{\partial^2 \phi_x}{\partial y^2}+(D_{12}+D_{66})\frac{\partial^2 \phi_x}{\partial x\partial y}-A_{45}\phi_x+D_{66}\frac{\partial^2 \phi_y}{\partial x^2}+2D_{26}\frac{\partial^2 \phi_y}{\partial x\partial y}+D_{22}\frac{\partial^2 \phi_y}{\partial y^2}-A_{44}\phi_y=-I_1\omega^2 v-I_2\omega^2\phi_y \tag{5}$$

where u, v, w, ϕ_x and ϕ_y are the unknown displacement components of middle surface of the plate. A_{ij}, B_{ij} and D_{ij} are the stiffness components. I_i are the mass inertias, ω is the natural frequency of free vibration.

$$A_{ij}=\sum_{k=1}^{N_L}\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}dz \tag{6}$$

$$B_{ij}=\sum_{k=1}^{N_L}\overline{Q}_{ij}^{(k)}\int_{z_k}^{z_{k+1}}zdz \tag{7}$$

$$D_{ij}=\sum_{k=1}^{N_L}\overline{Q}_{ij}^{(k)}\int_{z_k}^{z_{k+1}}z^2dz \tag{8}$$

$$I_0=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho dz,\quad I_1=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho zdz,\quad I_2=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho z^2dz \tag{9}$$

where ρ denotes the material density, N_L is total number of layer, z_k and z_{k+1} are the lower and upper z coordinates of the kth layer, $\overline{Q}_{ij}^{(k)}$ are the transformed elastic coefficients defined as [20].

The boundary conditions for an arbitrary edge with simply supported and clamped supported are as follows.

Simply supported:

$$\begin{aligned}&x=0,\quad a: u=v=w=\phi_y=0,\quad M_x=0\\&y=0,\quad b: u=v=w=\phi_x=0,\quad M_y=0\end{aligned} \tag{10}$$

Clamped:

$$u=v=w=\phi_x=\phi_y=0 \tag{11}$$

where

$$M_x=B_{11}\frac{\partial u}{\partial x}+B_{16}\frac{\partial u}{\partial y}+B_{12}\frac{\partial v}{\partial y}+B_{16}\frac{\partial v}{\partial x}+D_{11}\frac{\partial \phi_x}{\partial x}+D_{16}\frac{\partial \phi_x}{\partial y}+D_{16}\frac{\partial \phi_y}{\partial x}+D_{12}\frac{\partial \phi_y}{\partial y} \tag{12}$$

$$M_y=B_{12}\frac{\partial u}{\partial x}+B_{26}\frac{\partial u}{\partial y}+B_{22}\frac{\partial v}{\partial y}+B_{26}\frac{\partial v}{\partial x}+D_{12}\frac{\partial \phi_x}{\partial x}+D_{26}\frac{\partial \phi_x}{\partial y}+D_{26}\frac{\partial \phi_y}{\partial x}+D_{22}\frac{\partial \phi_y}{\partial y} \tag{13}$$

3. Meshless local radial point collocation method

Eqs. (1)–(5) and corresponding boundary conditions can be expressed in the following form:

$$\begin{aligned}&LU(X)=\omega^2U(X),\quad X\in\Omega\\&BU(X)=0,\quad X\in\partial\Omega\end{aligned} \tag{14}$$

where $\partial\Omega$ is the boundary of the problem domain Ω , L is a linear elliptic partial differential operator, B is a linear boundary operator. The solution of Eq. (14) can be approximated with a function $U^h(X)$ in the following form [15]:

$$U^h(X)\approx\sum_{i=1}^{n}\alpha_iR_i+\sum_{j=1}^{m}\beta_jP_j \tag{15}$$

where n is the number of nodes in the support domain, m is the number of terms of monomial, α_i and β_j are unknown coefficients, R_i is radial basis function, P_j is polynomial basis function.

Radial basis function used in this paper is multiquadric as follows:

$$R_i=\left((x-x_i)^2+(y-y_i)^2+(\alpha_c d_c)^2\right)^{0.5} \tag{16}$$

where x_i and y_i are coordinate of node i, α_c is shape parameter, d_c is the average distance of adjacent nodes.

Polynomial basis function used in this paper is as follows:

$$P^T=[1,x,y] \tag{17}$$

The number of terms of monomial $m = 3$.

The constraint condition is

$$P\alpha=0 \tag{18}$$

Eqs. (15) and (18) can be rewritten in matrix form by enforcing the interpolation passing through the value at all nodes in the supporting domain.

$$\begin{bmatrix}R & P^T\\ P & 0\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=G\eta=\begin{bmatrix}U_s\\ 0\end{bmatrix} \tag{19}$$

where

$$\begin{bmatrix}R & P^T\end{bmatrix}=\begin{bmatrix}R_1(r_1) & R_1(r_2) & \dots & R_1(r_n) & 1 & x_1 & y_1\\ R_2(r_1) & R_2(r_2) & \dots & R_2(r_n) & 1 & x_2 & y_2\\ \dots & \dots & \dots & \dots & \dots & \dots & \dots\\ R_n(r_1) & R_n(r_2) & \dots & R_n(r_n) & 1 & x_n & y_n\end{bmatrix} \tag{20}$$

$$\begin{bmatrix}P & 0\end{bmatrix}=\begin{bmatrix}1 & 1 & \dots & 1 & 0 & 0 & 0\\ x_1 & x_2 & \dots & x_n & 0 & 0 & 0\\ y_1 & y_2 & \dots & y_n & 0 & 0 & 0\end{bmatrix} \tag{21}$$

$$\eta^T=[\alpha_1,\alpha_2,\dots,\alpha_n,\beta_1,\beta_2,\beta_3] \tag{22}$$

where

$$R_i(r_j)=\left((x_i-x_j)^2+(y_i-y_j)^2+(\alpha_c d_c)^2\right)^{0.5} \tag{23}$$