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Identification of the elastic and damping properties in sandwich structures with a low core-to-skin stiffness ratio

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ABSTRACT

A mixed numerical–experimental identification procedure for estimating the storage and loss properties in sandwich structures with a soft core is developed. The proposed method uses at the experimental level a precise measurement setup with an electro-dynamic shaker and a scanning laser interferometer, and at the computational level an original structurally damped shell finite element model derived from the higher-order shear deformation theory with piecewise linear functions for the through-the-thickness displacement. The parameter estimation is derived from adequate objective functions measuring the discrepancy between the experimental and numerical modal data. Through a sensitivity analysis it is shown that for sandwich structures with a soft core only one specimen is required for characterizing the dominant properties of both the core and the skins. The procedure is then applied to two test cases for which all the influent elastic properties and the major damping parameters could be estimated with a fairly good precision.

1. Introduction

Sandwich structures with composite face sheets and lightweight core are intensively used in engineering fields where high bending stiffness and strength combined with a reduced weight is of prime importance. Typical applications where this construction type is attractive include among others ship hulls, aircraft wings, racing car frames or sports goods. Despite their attractiveness, sandwich constructions can only be used with confidence in engineering applications if their elastic and damping properties are thoroughly characterized. The high orthotropy and heterogeneity of these materials, as well as their specific production process, increase however the difficulty to identify their constitutive parameters in comparison to multilayered composites or usual materials such as metals, so that a powerful and effective parameter identification method for sandwich structures is highly desirable.

Although damping is of prime importance in noise and vibration control or in impact resistance and fatigue analysis of composite materials, the identification of the dissipative properties in multilayered structures and specifically in sandwich constructions has not received the same attention as the characterization of the elastic parameters. This issue is however particularly challenging as the damping properties can be tailored in a wide range by modifying the constituent materials, lamina thickness, ply orientation, stacking sequence or fibre fraction in multilayered structures and, in addition, by changing the skin thickness or core depth in sandwich constructions.

The estimation of the internal damping parameters in sandwich plates and shells – sometimes simultaneously with the elastic properties – is studied only recently. Based generally upon the complex stiffness modulus approach, finite element models relying on the first- or higher-order shear deformation shell theory [1–4], the layer-wise shell approach [5–10] or a full 3D formulation [11] have been developed for simulating the modal behaviour of damped sandwich plates or shells. Unfortunately, in most of these works, the damping loss factors or the specific damping capacities of the constituent materials are considered as given (quantities arbitrarily chosen, measured separately or found in the open literature). The investigations related in these publications are more focused on the analysis of the damping characteristics in sandwich structures than their true identification. A few [2,6–8] of the aforementioned works are however based upon mixed numerical–experimental characterization procedures, which have shown their efficiency in the identification of the elastic and damping properties in dissipative multilayered structures [12–18]. Constituting undoubtedly the most efficient techniques for estimating the internal damping factors in passively damped sandwich constructions, these methods are able to evaluate, from modal or harmonic vibration measurements, the internal damping loss factors and elastic properties in sandwich plates or shells modelled with various numerical formulations. In these identification procedures,

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experimental modal parameters (natural frequencies, mode shapes and modal loss factors) [2,8] or frequency response functions [6,7], measured on a plate or shell specimen, are compared to the equivalent predictions extracted from a finite element plate or shell model. The complex elastic and damping material properties chosen in the numerical model for the specimen under investigation are then updated iteratively in an optimisation loop by minimizing a given residual between the numerical predictions and the experimental results.

The current paper presents a mixed numerical–experimental characterization method based on the harmonic response of sandwich plates and shells with a low core-to-skin stiffness ratio. The technique is founded on the minimization of the discrepancies between modal quantities computed with a highly accurate orthotropic shell finite element model with adjustable elastic and structural damping properties and the corresponding experimental data measured with a precise measurement setup formed by a scanning laser vibrometer and a shaker-based excitation. By choosing appropriate error functionals which take into account not only the differences between the predicted and measured natural frequencies and modal loss factors but also the differences in amplitude, position or orthogonality between the simulated and measured complex mode shapes, an efficient identification of the constitutive properties in composite sandwich structures can be developed.

2. Modal numerical–experimental identification procedure

The proposed mixed numerical–experimental identification procedure for estimating the elastic and damping properties of sandwich materials is derived from a similar technique [18] developed by the authors for characterizing the elastic and damping parameters of multilayered structures.

The method is based upon the following main steps: (a) the development of a finite element shell model for the numerical modal simulation of damped sandwich composites, (b) the design of an experimental setup for measuring the modal response (complex mode shapes, natural frequencies and modal loss factors) of sandwich plates or shells, and (c) the elaboration of an optimisation procedure for finding the most adequate elastic and dissipative constitutive parameters minimizing an error functional containing appropriate error norms which characterize the discrepancies between the numerical predictions and the experimental results. These three main tasks are described in the next subsections.

2.1. Finite element model

On the computational level, different finite element models have been investigated in order to define the most suitable numerical formulation for modelling sandwich structures. Since the sandwich plates considered were made of face sheets in a rigid orthotropic material such as carbon/epoxy embedding a softer core with a low density like foam or honeycomb, the difficulty in modelling such type of material lies in the correct representation of the discontinuity of the through-the-thickness displacement derivative at the interfaces of the sandwich components. As the displacement in the thickness direction shows a well pronounced zigzag variation due to the strong difference of the elastic properties between the face sheets and the core of sandwich constructions, the true displacement field cannot be well estimated when using a traditional multilayered shell finite element formulation based upon the first- or higher-order shear deformation theory.

The finite element model used in this paper for the characterization of the elastic and dissipative properties in sandwich plates and shells is derived from a shell formulation based on a variable p-order shear deformation theory including also piecewise linear functions for the through-the-thickness displacement [19]. The present formulation could perhaps be viewed as rather outdated in comparison to layer-wise models. However, in an identification procedure, only the global structural behaviour like modal information is needed, so that the model chosen is computationally more efficient than the layer-wise approach which is effective when the local structural behaviour such as through-the-thickness stress and strain distribution is required.

Defined in the natural co-ordinate system $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$, the sandwich plate or shell element (Fig. 1a) is assumed to be composed of ${}^e m$ plies, ξ_3 being the thickness co-ordinate normal to the mid-surface (ξ_1–ξ_2). For sake of simplicity, the ratio ${}^e\theta_k$ ($k = 1, 2, \ldots, {}^e m$) between the thickness ${}^e t^i_{3k}$ of the kth ply at node i and the element thickness ${}^e t^i_3$ at the same nodal point is considered constant per layer and is given by

$$ {}^e\theta_k = \frac{{}^e t^i_{3k}}{{}^e t^i_3} \quad (i = 1, 2, \ldots, {}^e n;\ k = 1, 2, \ldots, {}^e m) \tag{1} $$

where ${}^e n$ denotes the total number of nodal points on the element mid-surface. The normalized transverse co-ordinate of each layer $\zeta = [-1, +1]$ is related to the variable ξ_3 by the following expression (Fig. 1a)

$$ \xi_3 = -1 + 2\sum_{j=1}^{k-1} {}^e\theta_j + (1+\zeta)\,{}^e\theta_k \tag{2} $$

which holds for every nodal point due to the proportionality law (1) adopted for the thickness of each layer. It should be added that the total number of layers ${}^e m$ is in general chosen equal to three for sandwich constructions with one core and two sheets, but a higher number of plies per sheet or for the core is conceivable.

The approximate generalized displacement vector ${}^e\mathbf{u}^h$ of order p for the sandwich shell element ${}^e\Omega$ (Fig. 1a) is then expressed as

$$ {}^e\mathbf{u}^h(\xi, t) = \sum_{i=1}^{{}^e n} {}^a h_i(\xi_1, \xi_2) \left[{}^e\mathbf{q}_i^{(0)}(t) + \xi_3 {}^e\mathbf{q}_i^{(1)}(t) + (-1)^k \zeta {}^e\mathbf{q}_i^Z(t) + \frac{\xi_3^2 {}^e\mathbf{q}_i^{(2)}(t)}{2!} + \cdots + \frac{\xi_3^p {}^e\mathbf{q}_i^{(p)}(t)}{p!} \right] \tag{3} $$

where t denotes time and ${}^a h_i(\xi_1, \xi_2)$ ($i = 1, 2, \ldots, {}^e n$) is the ith polynomial shape function of the master element ${}^a\Omega$, whereas ${}^e\mathbf{q}_i^{(j)}(t) = \left\{ {}^e q^{(j)}_{1,i}, {}^e q^{(j)}_{2,i}, {}^e q^{(j)}_{3,i} \right\}^{\mathrm{T}}$ ($i = 1, 2, \ldots, {}^e n$; $j = 0, 1, \ldots, p$) are the nodal degrees-of-freedom defined as the jth derivatives of the displacement approximation with respect to the shell thickness local co-ordinate ξ_3 at node i and ${}^e\mathbf{q}_i^Z(t) = \left\{ {}^e q^Z_{1,i}, {}^e q^Z_{2,i}, {}^e q^Z_{3,i} \right\}^{\mathrm{T}}$ ($i = 1, 2, \ldots, {}^e n$) denotes the vector containing the degrees-of-freedom at node i corresponding to the piecewise linear functions. Even if Eq. (3) contains simultaneously the local co-ordinate ξ_3 and the local layer co-ordinate ζ, no displacement inconsistency is created in the formulation. It can be seen (Fig. 1b) that the combination of the piecewise linear functions with the p-order polynomials induces displacement approximations through the thickness which are piecewise functions of order p. It should also be observed that the components in ${}^e\mathbf{q}_i^Z$ ($i = 1, 2, \ldots, {}^e n$) have a magnitude that is layer independent, which ensures that the intrinsic equivalent single layer character of the model is preserved.

The $3(p+2)$ degrees-of-freedom at nodal point i can be collected into the vector ${}^e\mathbf{q}_i$

$$ {}^e\mathbf{q}_i(t) = \left\{ {}^e q^{(0)}_{1,i}, {}^e q^{(0)}_{2,i}, {}^e q^{(0)}_{3,i}, {}^e q^{(1)}_{1,i}, {}^e q^{(1)}_{2,i}, {}^e q^{(1)}_{3,i}, {}^e q^{Z}_{1,i}, {}^e q^{Z}_{2,i}, {}^e q^{Z}_{3,i}, {}^e q^{(2)}_{1,i}, {}^e q^{(2)}_{2,i}, {}^e q^{(2)}_{3,i}, \ldots, {}^e q^{(p)}_{1,i}, {}^e q^{(p)}_{2,i}, {}^e q^{(p)}_{3,i} \right\}^{\mathrm{T}} \tag{4} $$