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Neural network modelling for damage behaviour of composites using full-field strain measurements

H. Man *, G. Prusty

School of Mechanical and Manufacturing Engineering, J17, University of New South Wales, Sydney, NSW 2052, Australia

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ABSTRACT

This paper presents a neural network modelling method for damage behaviour of composite materials in conjunction with full-field strain measurements. The proposed method utilises the overall structural response of a laminate composite specimen to develop the constitutive model of a single ply unidirectional laminate. Based on an energy principle, a performance function for training the neural networks is derived in terms of the applied external work and the induced strain energy. This allows the proposed method to develop the neural networks without the presence of stress information that is not necessarily obtainable in experiments with non-uniform deformation. The use of neural networks also enables the proposed method to model the damage behaviour without the constraints on the parameter space, such that a more representative model is developed for the actual material behaviour. An example of tailoring the proposed method to model the in-plane shear damage behaviour of a carbon fibre reinforced plastic (CFRP) laminate is demonstrated as well as its numerical validation. The practical application of the proposed method to multi-axial damage-related nonlinear behaviour of composite is presented using the experimental data obtained from a tensile test with an open-hole specimen.

1. Introduction

Different types of polymer and metal matrix composites, reinforced with continuous fibres, have been used in a broad range of engineering applications in the past decades [1,2]. Their anisotropic nature gives high degree of flexibility for structural design and allows a specific stiffness requirement to be achieved, albeit it also complicates the overall structural responses. The constitutive models of composite materials, therefore, become significantly important in order to conduct analysis that shows the realistic structural responses, including the complex damage-related behaviour [3,4].

The damage behaviour of fibre reinforced composites is conventionally modelled through analysing various damage mechanisms, such as matrix microcracking, fibre/matrix debonding and fibre breakage [5,6]. The models developed generally consist of a number of physical parameters that assist the constitutive model to represent the corresponding damage behaviour [7–10]. In conjunction with full-field measurements, Chalal et al. applied the so-called virtual fields method to identify the parameters of a damage model and their work demonstrated the advantages of characterising the damage behaviour of materials based on the entire structural responses. However, as pointed out by Mast et al. [11], the combination of various damage mechanisms makes the conventional damage models impossible to reflect the actual constitutive responses of the entire structure. As an alternative approach, they quantified the damage in the composite structures on a continuum basis using dissipated energy [11–13]. They utilised an in-plane loader system to generate a significant amount of experimental data to compute the dissipated energy of the whole specimen to represent the degree of damage. The work of Michopoulos et al. [14] demonstrated the same approach with the use of a six degree-of-freedom loading system and full-field strain measurements to quantify the dissipated energy of a composite specimen. Huang et al. [15] presented a comparable work based on dissipated energy to evaluate the damage of FRC as well as its failure conditions. Both works further emphasised the potential of characterising composite damage solely based on the energetic considerations of failure.

The aforementioned past works contributed to the insight of modelling the damage behaviour of composites, through those the developed models were represented by certain parametric forms regardless the representations of damage. As pointed out by Mast et al. [11], the failure behaviour of composites requires a large parameter space for its mathematical representation due to the complex damage phenomena. This, indeed, can be accomplished by using neural network modelling that was proven to provide equivalent functionalities as those explicitly defined models with minimal error [16–18]. Neural network-based constitutive

* Corresponding author.
E-mail address: h.man@unsw.edu.au (H. Man).

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models conventionally require both stress and strain data for the model development [19]. This requirement, however, is not necessarily achievable when analysing the deformation of the entire specimen since the stresses are not measurable.

This paper presents a neural network modelling method for damage behaviour of composite materials with the use of full-field strain measurements. The proposed method first derives a performance function based on an energy principle and is expressed in terms of the strain energy and external work increments. Along with the full-field strain measurement as well as the boundary displacement and force measurement of a load test, the performance function is used to develop the neural network to represent the damage-related nonlinear behaviour without the need of stress–strain pairs. The elimination of the requirement of stress facilitates the model development by only using a single load test that generates non-uniform strain fields embedded with multi-directional material responses.

Section 2 of the paper presents the proposed energy-based implicit modelling method generalised for constitutive relations of anisotropic materials. Section 3 presents the application for modelling the in-plane shear damage behaviour of a unidirectional laminate and is followed by the validation using a numerical example. Section 5 presents the experimental validation for the proposed method. Finally, a summary of the works presented in this paper is given in Section 6.

2. Energy-based neural network modelling

Energy-based implicit modelling utilised the discrepancy between the applied external work and the induced strain energy of a specimen to develop the constitutive model. It starts from the principle of minimum total potential energy for a conservative system, in which the equilibrium state of a deformed elastic body remains stable when the total potential energy of the body is minimised:

$$\delta\Pi = \delta U - \delta W, \tag{1}$$

where Π is the total potential energy, U and W are the induced strain energy and applied external work respectively and δ denotes the variational operator of an infinitesimal step. With the assumption that this principle is valid in the discrete time domain, an equilibrium equation can be expressed as

$$\Delta U^k = \Delta W^k, \tag{2}$$

where $k \in 1,2,\ldots,K$ represents the time step and Δ denotes the increment operator. The increment of the strain energy at the kth step is approximated as

$$\Delta U^k = \frac{1}{2}\int_{V^k}(\boldsymbol{\sigma}^k + \boldsymbol{\sigma}^{k-1})^{\mathrm{T}}\Delta\boldsymbol{\varepsilon}^k dV^k, \tag{3}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the column matrices of stress and strain respectively and V^k is the volume of the deformed body. With further assumption that the external work is applied on a constant surface, the variation of the external work at the kth step is represented as

$$\Delta W^k = \frac{1}{2}\int_{S}(\mathbf{P}^k + \mathbf{P}^{k-1})^{\mathrm{T}}\Delta\mathbf{u}^k dS, \tag{4}$$

where S is the surface and $\mathbf{P}$ and $\Delta\mathbf{u}$ are column matrices corresponding to the forces and the variations of the displacements, respectively. Both energy components of Eq. (2) are, therefore, represented in terms of the strain and external forces/displacements that can be practically measured in experiments [20] as well as the stress that is replaced by a neural network-based constitutive model.

2.1. Association of neural network to global coordinate system

The proposed method applies the neural networks to represent the stress–strain constitutive relations in the formulations. A neural network for the stress–strain relations in the material coordinate system is given as

$$\boldsymbol{\sigma}_M = \phi(\boldsymbol{\varepsilon}_M; \Omega), \tag{5}$$

the subscript M represents the material coordinate system and Ω is a set of non-physical parameters inside the neural networks with a finite but large number [21], and is denoted as $\Omega = \{\boldsymbol{\omega}_{\{1\}}, \boldsymbol{\omega}_{\{2\}}, \ldots, \boldsymbol{\omega}_{\{L\}}, \mathbf{b}_{\{1\}}, \mathbf{b}_{\{2\}}, \ldots, \mathbf{b}_{\{L\}}\}$ with $\boldsymbol{\omega}_{\{i\}}$ as the network weights and $\mathbf{b}_{\{i\}}$ as the bias of the ith layer. Note that Eq. (5) is able to represent any path independent behaviour for both isotropic and anisotropic materials. However, for anisotropic materials, it is required to express the above neural network in terms of the stress and strain in the global coordinate system in order to apply the experimental measurements for the neural network development. For an anisotropic material with θ as the transformation angles between the material and global coordinate systems, the relation of the strain in the two coordinate systems is expressed as

$$\boldsymbol{\varepsilon}_M = \mathbf{T}_{\varepsilon}(\theta)\boldsymbol{\varepsilon}, \tag{6}$$

where $\mathbf{T}_{\varepsilon}(\theta)$ is the transformation matrix for strain. Substitution of Eq. (6) into Eq. (5) yields

$$\boldsymbol{\sigma}_M = \phi(\mathbf{T}_{\varepsilon}(\theta)\boldsymbol{\varepsilon}; \Omega). \tag{7}$$

Similarly, the transformation of stress between the global coordinate system and the material coordinate system is expressed as

$$\boldsymbol{\sigma} = \mathbf{T}_{\boldsymbol{\sigma}}(\theta)^{-1}\boldsymbol{\sigma}_M. \tag{8}$$

Substitution of Eq. (7) into Eq. (8) subsequently yields

$$\boldsymbol{\sigma} = \mathbf{T}_{\boldsymbol{\sigma}}(\theta)^{-1}(\phi(\mathbf{T}_{\varepsilon}(\theta)\boldsymbol{\varepsilon}; \Omega)), \tag{9}$$

to represent the stress–strain in the global coordinate system. The coordinate transformation with the neural networks ensures that the formulations of the proposed method are applicable to material frame-indifference and also ensures that the experimental measurements in global coordinate system can be utilised for the neural network development.

2.2. Derivation of the energy-based performance function

Based on the energy principle and Eq. (2), a performance function is expressed as

$$F = \frac{1}{K}\sum_k \left\|\Delta U^k - \Delta W^k\right\|^2, \tag{10}$$

where F is defined as the performance measure for a set of experimental measurement obtained in discrete time step. The increment of total strain energy at the kth step is represented as a function of both stress and strain in the global coordinate system as shown in Eq. (3). By substituting the approximated stress from Eq. (9) into (3), the total strain energy increment becomes

$$\Delta U^k = \frac{1}{2}\int_{V^k}\left\{\mathbf{T}_{\boldsymbol{\sigma}}^{-1}(\theta)\left[\phi(\mathbf{T}_{\varepsilon}(\theta)\boldsymbol{\varepsilon}^k; \Omega) + \phi(\mathbf{T}_{\varepsilon}(\theta)\boldsymbol{\varepsilon}^{k-1}; \Omega)\right]\right\}^{\mathrm{T}}\Delta\boldsymbol{\varepsilon}^k dV^k. \tag{11}$$

Consider a specimen with constant thickness is divided to a number of elements, which correspond to the elemental strain measurement, ${}^e\mathbf{z}_{\varepsilon}^k$, the above equation becomes

$$\Delta\check{U}^k = \frac{1}{2}\sum_e {}^eA^k t\left[\mathbf{T}_{\boldsymbol{\sigma}}^{-1}(\theta)\phi^*\left({}^e\mathbf{z}_{\varepsilon}^k, {}^e\mathbf{z}_{\varepsilon}^{k-1}, \mathbf{T}_{\varepsilon}(\theta); \Omega\right)\right]^{\mathrm{T}}\Delta^e\mathbf{z}_{\varepsilon}^k \tag{12}$$