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Buckling analysis of micro-/nano-scale plates based on two-variable refined plate theory incorporating nonlocal scale effects

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ABSTRACT

This article presents the buckling analysis of isotropic nanoplates using the two variable refined plate theory and nonlocal small scale effects. The two variable refined plate theory takes account of transverse shear effects and parabolic distribution of the transverse shear strains through the thickness of the plate, hence it is unnecessary to use shear correction factors. Nonlocal governing equations of motion for the nanoplate are derived from the principle of virtual displacements. The closed-form solution for buckling load of a simply supported rectangular nanoplate subjected to in-plane loading has been obtained by using the Navier's method. Numerical results obtained by the present theory are compared with available exact solutions in the literature. The effect of nonlocal scaling parameter, mode numbers and aspect ratios of the nanoplates on buckling load are investigated and discussed in detail in the present work. It can be concluded that the present theory, which does not require shear correction factor, is not only simple but also comparable to the first-order and higher order shear deformable theory.

1. Introduction

Nanoplates such as graphene [1], the two-dimensional (2D) counterpart of three-dimensional graphite, has attracted vast interests in solid-state physics, materials science, and nanoelectronics since it was discovered in 2004 as the first free-standing 2D crystal. Graphene is considered as a promising electronic material in post silicon electronics. However, large-scale synthesis of high quality graphene represents a bottleneck for the next generation graphene devices. Existing routes for graphene synthesis include mechanical exfoliation of highly ordered pyrolytic graphite (HOPG) [2], eliminating Si from the surface of single crystal SiC [3], depositing graphene at the surface of single crystal [4] or polycrystalline metals [5], and various wet-chemistry based approaches [6,7]. However, up to now no methods have delivered high quality graphene with large area required for application as a practical electronic material.

A great deal of research has been conducted to explore the promising properties of the single-layered graphene sheets (SLGSs) after appearance of the new method of graphene sheet preparation [8–10]. Furthermore, Katsnelson and Novoselov [11] have explored the unique electronic properties of the SLGSs. They have stated that the graphene sheet is an unexpected bridge between condensed matter physics and quantum electrodynamics. Moreover, Meyer et al. [12] have achieved the ability of distinguishing between single- and multi-layered graphene sheets by analyzing electron diffraction. On the other hand, Bunch et al. [13] have reported the experimental results of using electromechanical resonators made from suspended single- and multi-layered graphene sheets.

Because of the unique electrical, mechanical and thermal properties enable the nanostructures (such as graphene, carbon nanotube, nanorod, and nanofibre) to be used for the development of superconductive devices for micro-electromechanical system (MEMS) and nano-electromechanical system (NEMS) applications. Conducting experiments with nanoscale size specimens is both difficult and expensive. Hence, development of appropriate mathematical models for nanostructures is an important issue concerning the application of nanostructures. The modeling for the nanostructures is classified into three main categories. The approaches are atomistic [14,15], continuum [14,15] and hybrid atomistic-continuum mechanics [16–18]. The above atomic methods are limited to systems with a small number of molecules and atoms and therefore restricted to the study of small-scale modeling. In order to carry out analysis for a large-sized atomic system, other powerful and effective models for the analysis are needed. Continuum mechanics approach is less computationally expensive than the former two approaches. Further, it has been found that continuum mechanics results are in good agreement with those obtained from atomistic and hybrid approaches.

Nanotechnologies small scale makes the applicability of classical or local continuum models, such as beam, shell and plate models, questionable. Classical continuum models do not admit

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Table 1
Non-dimensional critical buckling load.

g (nm)	EBT Ref. [62]	RBT Ref. [62]	HSDT Ref.[61]	FSDT (k = 2/3)	FSDT (k = 5/6)	FSDT (k = 1)	Present NLTVRPT
L/h = 20							
0.0	9.8696	9.8067	9.8067	9.8005	9.8066	9.8108	9.8067
0.5	9.4055	9.3455	9.3455	9.3393	9.3453	9.3496	9.3455
1.0	8.9830	8.9528	8.9528	8.9466	8.9526	8.9569	8.9528
1.5	8.5969	8.5421	8.5421	8.5359	8.5419	8.5462	8.5421
2.0	8.2426	8.1900	8.1900	8.1838	8.1898	8.1941	8.1900
L/h = 100							
0.0	9.8696	9.8671	9.8671	9.8668	9.8671	9.8674	9.8671
0.5	9.4055	9.4031	9.4031	9.4029	9.4031	9.4033	9.4031
1.0	8.9830	8.9807	8.9807	8.9804	8.9807	8.9810	8.9807
1.5	8.5969	8.5947	8.5947	8.5946	8.5947	8.5949	8.5947
2.0	8.2426	8.2405	8.2405	8.2402	8.2405	8.2409	8.2405

intrinsic size dependence in the elastic solutions of inclusions and inhomogeneities. At nanometer scales, however, size effects often become prominent, the cause of which needs to be explicitly addressed due to an increasing interest in the general area of nanotechnology [19]. Sun and Zhang [20] indicated the importance of a semi-continuum model in analyzing nanomaterials after pointing out the limitations of the applicability of classical continuum models to nanotechnology. In their semi-continuum model for nanostructured materials with plate like geometry, material properties were found completely dependent on the thickness of the plate structure contrary to classical continuum models. The modeling of such a size-dependent phenomenon has become an interesting research subject in this field [21–23]. It is thus concluded that the applicability of classical continuum models at very small scales is questionable, since the material microstructure, such as lattice spacing between individual atoms, becomes increasingly important at small size and the discrete structure of the material can no longer be homogeneities into a continuum. Therefore, continuum models need to be extended to consider the scale effect in nanomaterial studies. This can be accomplished through proposing nonlocal continuum mechanics models.

Nonlocal elasticity theory [24–29] was proposed to account for the scale effect in elasticity by assuming the stress at a reference point to be a function of strain field at every point in the body. This way, the internal size scale could be simply considered in constitutive equations as a material parameter Only recently has the nonlocal elasticity theory been introduced to nanomaterial applications. As the length scales are reduced, the influences of long-range interatomic and intermolecular cohesive forces on the static and dynamic properties tend to be significant and cannot be neglected. The classical theory of elasticity being the long wave limit of the atomic theory excludes these effects. Thus the traditional classical continuum mechanics would fail to capture the small scale effects when dealing in nano structures. The small size analysis using local theory over predicts the results. Thus the consideration of small effects is necessary for correct prediction of micro/nano structures. Chen et al. [30] that the nonlocal continuum theory based models are physically reasonable from the atomistic viewpoint of lattice dynamics and molecular dynamics (MD) simulations. Peddieson et al. [31] applied nonlocal elasticity to formulate a nonlocal version of the Euler–Bernoulli beam model and concluded that nonlocal continuum mechanics could potentially play a useful role in nanotechnology applications.

In wave mechanics of nanostructures, one important outcome of the nonlocal elasticity is the realistic prediction of the dispersion curve i.e., frequency-wavenumber/wavevector relation. As shown in Eringen [24], the dispersion relation

$$\frac{\omega}{C_1 k} = (1 + (e_0 a)^2 k^2)^{-1/2} \tag{1}$$

where $e_0 a$ nonlocality parameter, closely matches with the Born-Karman model dispersion

$$\frac{\omega a}{C_1} = 2 \sin\left(\frac{ka}{2}\right) \tag{2}$$

when $e_0 = 0.39$ is considered. However, among the two natural conditions at the mid-point and end of the first Brillouin zone:

$$\left.\frac{d\omega}{dk}\right|_{k=0} = C_1; \quad \left.\frac{d\omega}{dk}\right|_{k=\frac{\pi}{a}} = 0 \tag{3}$$

these relations satisfy only the first one. It was suggested that two-parameter approximation of the kernel function will give better results. This is reiterated by Lazar et al. [66] that one parameter (only $e_0 a$) nonlocal kernel will never be able to model the lattice dynamics relation and it is necessary to use the bi-Helmholtz type equation with two different coefficients of nonlocality to satisfy all the boundary conditions.

It is to be noted that the simple forms of the group and phase velocities that exist for isotropic materials permitted to tune the nonlocality parameters so that the lattice dispersion relation is matched. Further, by virtue of the Helmholtz decomposition, only one-dimensional Brillouin zone needs to be handled. Although the general form of the boundary conditions, i.e., group speed is equal to phase speed (at $k = 0$) or zero (at $k = \pi/a$), is still applicable, the expressions are difficult to handle. This is because, the Brillouin zone is really a two-dimensional region where four boundary conditions are involved.

Various size-dependent continuum theories which capture small scale parameter such as couple stress elasticity theory [32], strain gradient theory [33], modified couple stress theory [34] are reported. These modified continuum theories are being used for the analysis of small scale structures. However, the most reportedly used continuum theory for analyzing small scale structures is the nonlocal elasticity theory initiated by Eringen [24]. Using this nonlocal elasticity theory, some drawbacks of the classical continuum theory can be efficiently avoided and size-dependent phenomena can be satisfactorily explained. In nonlocal elasticity theory the small scale effects are captured by assuming the stress components at a point is dependant not only on the strain components at the same point but also on all other points in the domain [24,35].

In the literature a great deal of attention has been focused on studying the buckling behavior of one-dimensional nanostructures using nonlocal elasticity theory. These nanostructures include nanobeams, nanorods and carbon nanotubes. On the contrary no work appears related to the buckling of biaxially compressed nanoplate based on two-variable refined plate theory. However some studies using nonlocal elasticity theory on mechanical behavior of isotropic nanoplates are recently reported by Murmu and Pradhan [36–38] and by Duan and Wang [39]. Recently Sakhaee-Pour