Composite Structures 93 (2011) 3200–3208

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Viscoelastic analysis of FRP strengthened reinforced concrete beams

Chao Zhang, Jialai Wang [*]

Department of Civil, Construction, and Environmental Engineering, The University of Alabama, Tuscaloosa, AL 35487, United States

ARTICLE INFO

Article history:
Available online 21 June 2011

Keywords:
FRP-strengthening
Interface stress
Viscoelastic analysis
Long-term behavior
Stress redistribution
Creep

ABSTRACT

External bonding of FRP plates or sheets has become a popular method for strengthening reinforced concrete structures. Stresses along the FRP-concrete interface are critical to the effectiveness of this technique because high stress concentration along the FRP-concrete interface can lead to the FRP debonding from the concrete beam. Although the short-term stress distribution along the FRP-concrete interface has been studied extensively, very few studies have been conducted on the long-term stress distribution, which closely simulates the behavior of the structure during the service-life. In this study, we develop a viscoelastic solution for the long-term interface stress distribution in a FRP plate strengthened reinforced concrete beam. In this solution, the RC beam and the FRP plate are modeled as elastic materials; while the adhesive layer is modeled as a viscoelastic material using the Standard Linear Solid model. Closed-form expressions of the interface stresses and deflection of the beam are obtained using Laplace transform and calculated using the Zakian's numerical method. The validation of this viscoelastic solution is verified by finite element analysis using a subroutine UMAT based on the Standard Linear Solid model.

1. Introduction

Fiber reinforced polymers (FRPs) have emerged as important structural materials in the last three decades [1]. Among their many applications in civil infrastructure, retrofitting/rehabilitating reinforced concrete (RC) structures is most popular due to many advantages of FRP, such as high corrosion resistance, high strength-to-weight ratio, and easy of handling. In this application, FRP plates/fabrics/strips are either externally bonded (EB) [2] or near-surface mounted (NSM) on RC structures [3]. To improve the strengthening efficiency, prestress can be applied to FRPs [4]. This technique has evolved into one of primary techniques to address the deterioration of civil infrastructure system caused by severe environmental exposures, natural extreme events, excessive use, and intentional attacks. Stresses along the FRP-concrete interface are critical to the success of this technique because the high interface stress concentration can lead to debonding along the FPR-concrete interface, which has been shown to be one of the most common failure modes of the FRP-strengthened RC structures. Extensive studies have been conducted, and various models have been proposed to estimate the stresses along the FRP-concrete interface [5]. A comprehensive review on these studies was given by Smith and Teng [5] and a recent study by Wang and Zhang [6]. However, all these existing studies are limited to the elastic analysis, which only gives the instant response of the structures.

Epoxy, the most widely used adhesive in bonding FRP, exhibits viscoelastic properties [7–11]. Its material properties vary with time under different situations, especially in the regions of high stress concentration. Such variation of material properties can induce redistributions of stresses and additional deformation, which could be significant during the service life of the structure and cause potential failure of the strengthening, as demonstrated recently by Meshgin et al. [12]. Meshgin et al. [12] found that the creep of epoxy could result in failure at the interfaces due to the combined effect of relatively high shear stress to ultimate shear strength ratio and a thick layer of epoxy. For this reason, the time-dependent behavior of the FRP strengthened concrete structures has become the focus of a number of recent studies [12–18] both experimentally and numerically. All of these studies show that the concrete-FRP interface exhibits significant time-dependent behavior, and that the shear stress to the shear strength ratio within the adhesive layer is a primary factor affecting the long-term behavior of the FRP-concrete interfaces [18].

Several rheological models were proposed to simulate the creep behaviors observed in the tests [12,13,17,18]. Based on these rheological models, numerical methods [13,18] have been proposed to simulate the time-dependent behaviors of the FRP-strengthened RC structures. Numerical methods are usually very time-consuming in simulating the time-dependent behavior of structures because sufficient small step must be used to avoid error accumulation. Analytical solutions are much more efficient.

The existing analytical solutions of the interface stresses of FRP-strengthened RC beams [5,19–24] are based on the classical adhesively bonded joint model of Goland and Reissner [25] (G–R model).

---

[*] Corresponding author. Tel.: +1 205 348 6786; fax: +1 205 348 0783.
E-mail address: jwang@eng.ua.edu (J. Wang).

0263-8223/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2011.06.006

In this model, the strengthened beam and the FRP plate were modeled as two beams connected by a linear elastic adhesive layer, which is modeled as a layer of continuously distributed spring with shear and normal stiffnesses. However, all these analytical solutions are limited to elastic case. Delale and Erdogan [26] developed a viscoelastic solution for a symmetric adhesive joint by treating two adherends as elastic simple beams and the adhesive as a linearly viscoelastic spring. Mirman and Knecht [8] proposed another simple viscoelastic model to study the creep behavior of the adhesive layer, in which the peeling stress and the creep deformation of the adhesive layer were ignored.

So far no rigorous viscoelastic solutions for the interface stresses and deflection of the FRP-strengthened RC beams are available. The major objective of this research is to fill this gap through developing viscoelastic solutions of the interface stresses and the deflection of a FRP-strengthened RC beam. These new solutions can be used as an efficient tool to evaluate the long-term behavior of the FRP-strengthened RC beams.

2. Viscoelastic model of a FRP plate strengthened RC beam

Consider a simply supported RC beam (beam 1) with thickness of h_1 strengthened by a thin FRP plate (beam 2) with thickness of h_2 through external bonding with a thin adhesive layer with thickness of h_0, as shown in Fig. 1. A uniform load with magnitude of q is applied to the RC beam. Since the creep of the concrete and the FRP plate occurs within a much longer time period compared with the adhesive [15], only the adhesive layer is modeled as viscoelastic material in this study. Both the FRP plate and the RC beam are modeled as elastic material. It should be pointed out that this simplified model is only a first step to understand the adhesive creeping effect of the FRP-strengthened RC beams since the creep behaviors of both the FRP plate and the RC beams are not considered.

A notable recent development in the FRP strengthening technique is that the FRP plate is prestressed before bonding to the RC beam. This technique can make better use of the tensile strength of the FRP plate [27] and reduce the crack development in the RC beam [28]. Although a number of experiments have been conducted to study the short-term behaviors of the RC beams strengthened by this technique [29–31], their long-term behaviors have not been addressed sufficiently, especially the prestress loss due to the creep of adhesive layer, which is critical to the success of the prestressing technique. To make this study also applicable to the prestressed FRP strengthened RC beams, a prestress of N_0 is applied to the FRP plate in the formulation thereafter. This prestress is applied as following: First the FRP plate is pretensioned; then the prestressed FRP plate is bonded to the tension side of the RC beam; finally, cut the FRP plate at two far ends once the adhesive achieved its fully bonding strength.

2.1. Shear deformable beam theory

Both the RC beam and the FRP plate are modeled as Timoshenko beams, beam 1 and 2, respectively. The displacement fields can then be written as

$$\begin{aligned} &U_1(x_1,z_1,t)=u_1(x,t)+z_1\phi_1(x,t);\\ &U_2(x_2,z_2,t)=u_2(x,t)+z_2\phi_2(x,t)-u_0(x,t)\\ &W_i(x_i,z_i,t)=w_i(x,t), \end{aligned} \tag{1}$$

where subscript i = 1, 2, representing the beams *1* and *2* in Fig. 1. The x_i and z_i axels are local coordinates and x_i is located at the neutral axis of the beam i and $x_1 = x_2 = x \cdot u_i(x,t)$, $\phi_i(x,t)$, and $w_i(x,t)$ are the axial displacement, rotation and deflection at the neutral axis of beam i. $u_0(x,t)$ is the initial displacement of the FRP plate before the pretension is released. $U_i(x_i,z_i,t)$ and $W_i(x_i,z_i,t)$ are the axial and transverse displacements of the beam i. The strains along the neutral axes of these two beams can be written as

$$\begin{aligned} &\varepsilon_1^0(x,t)=\frac{\partial u_1(x,t)}{\partial x},\quad \varepsilon_2^0(x,t)=\frac{\partial u_2(x,t)}{\partial x}-\frac{\partial u_0(x,t)}{\partial x},\\ &\kappa_i(x,t)=\frac{\partial \phi_i(x,t)}{\partial x},\quad \gamma_{ixy}(x,t)=\phi_i(x,t)+\frac{\partial w_i(x,t)}{\partial x}. \end{aligned} \tag{2}$$

The constitutive equations for the beam i can be written as:

$$\begin{aligned} &N_1(x,t)=C_1\frac{\partial u_1(x,t)}{\partial x},\\ &N_2(x,t)-N_0H(t)=C_2\left(\frac{\partial u_2(x,t)}{\partial x}-\frac{\partial u_0(x,t)}{\partial x}\right),\\ &M_i(x,t)=D_i\frac{\partial \phi_i(x,t)}{\partial x},\\ &Q_i(x,t)=B_i\left(\phi_i(x,t)+\frac{\partial w_i(x,t)}{\partial x}\right), \end{aligned} \tag{3}$$

where $N_i(x,t)$, $Q_i(x,t)$ and $M_i(x,t)$ are the resultant axial force, the transverse shear force, and the bending moment of the beam i, respectively. $H(t)$ is the Heaviside step function. N_0 is the prestress applied to the FRP plate. C_i, B_i and D_i are the axial, shear and bending stiffness coefficients of the beam i, respectively. For plain stress condition, we have

$$C_i=E_ibh_i,\quad D_i=E_ibh_i^3/12,\quad B_i=\kappa G_ibh_i, \tag{4}$$

where E_i and G_i are the longitudinal modulus and the transverse shear modulus, respectively; κ is the shear correction coefficient chosen as 5/6 in this study; b is the width of the beam. The bending stiffness of the RC beam D_1 can be modified to account for a cracked section.

Considering an infinitesimal free body diagram of the FRP-strengthened RC beam shown in Fig. 2, the following equilibrium equations can be established

$$\begin{aligned} &\frac{\partial N_1(x,t)}{\partial x}=b\tau(x,t),\\ &\frac{\partial M_1(x,t)}{\partial x}=Q_1(x,t)-\frac{h_1}{2}b\tau(x,t),\\ &\frac{\partial Q_1(x,t)}{\partial x}=b\sigma(x,t)+bqH(t), \end{aligned} \tag{5a}$$

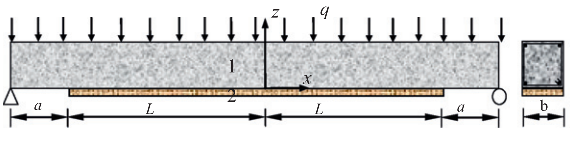

Fig. 1. An FRP strengthened RC beam.

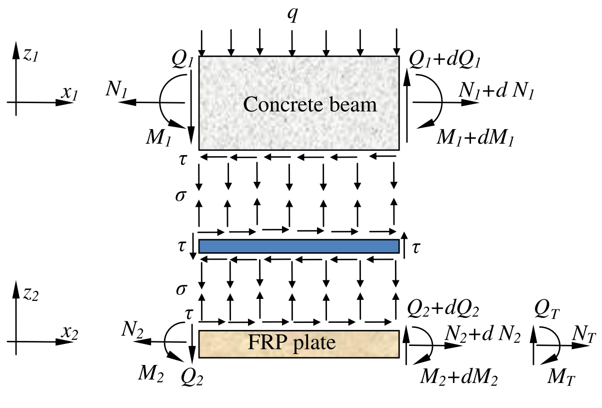

Fig. 2. Free body diagram of a FRP-strengthened RC beam.