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Interlaminar shear stresses around an internal part-through hole in a stretched laminated composite plate

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ABSTRACT

The equilibrium/compatibility method, which is a semi-analytical post-processing method, is employed for computation of hitherto unavailable through-thickness variation of interlaminar (transverse) shear stresses in the vicinity of the bi-layer interface circumferential re-entrant corner line of an internal part-through circular cylindrical hole weakening an edge-loaded laminated composite plate. A C^o-type triangular composite plate element, based on the assumptions of transverse inextensibility and layer-wise constant shear-angle theory (LCST), is utilized to first compute the in-plane stresses and layer-wise through-thickness average interlaminar shear stresses, which serve as the starting point for computation of through-thickness distribution of interlaminar shear stresses in the vicinity of the bi-layer interface circumferential re-entrant corner line of the part-through hole. The same stresses computed by the conventional equilibrium method (EM) are, in contrast, in serious errors in the presence of the bi-layer interface circumferential re-entrant corner line singularity arising out of the internal part-through hole, and are found to violate the interfacial compatibility condition. The computed interlaminar shear stress can vary from negative to positive through the thickness of a cross-ply plate in the neighborhood of this kind of stress singularity.

1. Introduction

Advanced composite materials (e.g., graphite/epoxy, boron/epoxy, Kevlar/epoxy, graphite/PEEK, etc.) are replacing metallic alloys at an accelerated pace in the fabrication of structural components in both military (e.g., stealth fighter F-117A Nighthawk and B-2 bomber) and commercial (e.g., the forthcoming Boeing 787 Dreamliner) airplanes. This is because of many beneficial properties, such as higher strength-to-weight ratios, longer in-plane fatigue (including sonic fatigue) life, better stealth characteristics, enhanced corrosion resistance, and most significantly, the possibility of optimal design through the variation of stacking pattern, fiber orientation, and so forth, known as composite tailoring. These advantages notwithstanding, polymeric composite plate type structures are highly prone to transverse (or interlaminar) shear related fatigue failures, as a result of the matrix material being of relatively low shearing stiffness as compared to the longitudinal stiffness of the fibers. A reliable prediction of the response of these laminated plates must account for transverse (or interlaminar) shear deformation.

The issues concerning the weakening effects of through-thickness holes in both homogeneous isotropic as well as laminated composite plates are well understood in the literature [1–5]. In contrast, the problems of stress concentration in the vicinity of internal part-through holes weakening both homogeneous as well as laminated plates have till recently remained a virgin territory [6–9]. Embedded part-through holes are more realistic representations of internal flaws and damages. For example, part of the material inside one or more layers may be missing as a result of faulty manufacturing techniques, which may propagate during service with catastrophic consequences. The fact that these internal holes can cause severe cross-sectional warping resulting in interlaminar or transverse strains and stresses even in a stretched/compressed plate, and as a result may initiate shear crippling failure in compression or shear/delamination failure in tension in advanced composite laminates is a matter of serious concern to structural designers. Additionally, these interlaminar stresses in the neighborhood of such a hole in a stretched homogeneous isotropic as well as laminated anisotropic plate vary through the thickness, which brings three-dimensional effect even in a thin plate weakened by such an embedded part-through hole [6–9]. Finally, the effect of stress singularity in the neighborhood of the bi-layer interface circumferential re-entrant corner line of the part-through internal hole weakening a laminated plate is also of serious concerns.

Although reasonably accurate (finite element based) post-processing type methods for determination of interlaminar or

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Nomenclature

a, b	length and width, respectively, of a rectangular laminated plate
$[B_j^{(i)}]$	strain-nodal displacement relation matrix for the jth triangular layer-element belonging to the ith layer
$[C^{(i)}]$, $[G^{(i)}]$	in-plane and transverse elastic stiffness matrices of the ith orthotropic lamina
$c_{jk}^{(i)}$	components of the elastic stiffness matrix of the ith orthotropic lamina
$[D_j]$	integrated (through thickness) elastic stiffness matrix
$\{d_j^{(i)}\}$	nodal displacement vector of the jth triangular layer-element belonging to the ith layer
$\bar{d}_i$	distance from the bottom (reference) surface
E_{11}, E_{22}	Young's moduli of an orthotropic lamina in the direction of fibers and normal to the fibers, respectively
$\{F_j\}$	consistent load vector of the jth composite plate element
g_α, g_β	first fundamental quantities for orthogonal curvilinear coordinates for a flat plate
G_{12}	in-plane shear modulus of an orthotropic lamina
G_{13}, G_{23}	transverse shear moduli of an orthotropic lamina
h	total depth of an internal part-through hole
$[K_j]$	stiffness matrix for the jth composite plate element
ℓ_α, ℓ_β	direction cosines of normal with respect to α and β coordinates, respectively
N	total number of layers or laminae
q	applied uniform tension at the edge of a plate
r, θ, z	polar cylindrical coordinates
r_0	radius of an internal part-through circular cylindrical hole
t_i, t	thickness of the ith lamina and laminated plate, respectively
U	strain energy of a laminated plate
$\overline{U}$	strain energy per unit area
$U_I^{(i)}$	strain energy due to intra-laminar stresses and strains of the ith layer
$U_S^{(i)}$	strain energy due to interlaminar (transverse) shear stresses and strains of the ith layer
u_i	in-plane components of the displacement vector, $i = \alpha,\ \beta$
W	potential due to external conservative forces
w	transverse displacement component (deflection) of a plate
x, y, z	Cartesian coordinates in the length, width and thickness directions of a laminated plate
α, β	in-plane (orthogonal) curvilinear coordinates
$\gamma_{\alpha\beta}^{(i)}$	in-plane (engineering) shearing strain at a point inside the ith layer
$\gamma_{\alpha z}^{(i)}$, $\gamma_{\beta z}^{(i)}$	transverse (engineering) shearing strains at a point inside the ith layer
Δ_j	reference surface area of the jth element
$\varepsilon_k^{(i)}$	in-plane normal strains at a point inside the ith layer; $k = \alpha,\ \beta$
ν_{12}, ν_{13}, ν_{23}	major Poisson's ratios of an orthotropic lamina
Π	total potential energy functional
$\bar{\theta}_i$	fiber orientation angle of the ith lamina
$\bar{\sigma}_{nn}^{(i)}(z)$, $\bar{\tau}_{n\Gamma}^{(i)}(z)$, $\bar{\tau}_{nz}^{(i)}(z)$	applied stresses at a boundary distributed through the thickness of the ith layer
$\{\phi\}$	shape functions

transverse shear stresses in homogeneous and laminated plates and shells have been developed in the 1980's [10–15], a detailed review of the literature reveals that till recently the issue of determination of transverse shear stresses in the vicinity of an internal (or embedded) part-through hole weakening a plate subjected to all-round tension has remained unaddressed in the literature [16,17]. Majority of the finite element based post-processing approaches employ equilibrium equations of three-dimensional elasticity theory, referred to here as equilibrium method, the only exceptions being Chaudhuri and Seide [13], and Chaudhuri [14]. These authors have introduced a semi-analytical post-processing method, termed equilibrium/compatibility method, wherein both the equilibrium equations as well as interfacial compatibility conditions are satisfied in the point-wise sense. The latter approach has recently been employed for computation of hitherto unavailable through-thickness variation of transverse shear stresses in the vicinity of the circumferential re-entrant corner lines of internal part-through circular and elliptical cylindrical holes weakening edge-loaded isotropic plates [16,17]. A detailed review of the literature, however, reveals that the determination of interlaminar shear stresses in the vicinity of an internal (or embedded) part-through hole weakening a laminated anisotropic plate, subjected to all-round tension, has still remained unaddressed in the literature. The primary objective of the present investigation is to fill this important gap. In what follows, a curvilinear triangular element, based on the assumptions of transverse inextensibility and layer-wise constant shear angle (LCST), is employed as a starting point to determine the through-thickness distribution of interlaminar or transverse shear stresses in the vicinity of an embedded circular part-through hole, weakening a general cross-ply plate, using a method that satisfies the equilibrium as well as interfacial compatibility in the point-wise sense. In this connection, it may be noted that the stress singularity, in the neighborhood of the bi-layer interface circumferential re-entrant corner line of the internal part-through hole, is that of a bi-material wedge type singularity [18,19] of which the well-known free-edge stress singularity [20–23] is a special case.

2. Theoretical background and finite element formulation

Fig. 1 shows an N-layer laminated composite triangular element with its bottom surface designated as the reference surface. The curvilinear triangular element is ideally suited for analysis of a laminated anisotropic plate weakened by a part-through hole of arbitrary geometry (e.g., circular, elliptical, etc.). The small deformation of such weakened plates [6–8] can be analyzed using a finite element formulated on the basis of assumptions of transverse inextensibility and layer-wise constant shear angle [24–26]. The special case of a moderately thick plate weakened by a through hole [4] can be analyzed by a finite element formulation based on the Mindlin hypothesis [27,28].

The local or element coordinates are denoted by α, β and z, while the corresponding global or plate coordinates are represented by x, y and z. The curvilinear coordinates for the special case of circular geometry are given by

$$\alpha = r, \quad \beta = \theta, \quad g_\alpha = 1, \quad g_\beta = r. \tag{1}$$

The distribution of displacement components for the ith layer, based on the above assumptions, can be written as follows [24–26]: