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A sub-laminates FEM approach for the analysis of sandwich beams with multilayered composite faces

M. Degiovanni *, M. Gherlone, M. Mattone, M. Di Sciuva

Politecnico di Torino, Aerospace Engineering Department (DIASP), Corso Duca degli Abruzzi, 24 – 10129 Torino, Italy

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ABSTRACT

A finite element approach based on the Hermitian ZigZag theory and on the sub-laminates concept is proposed for the static analysis of sandwich structures. The accuracy of the model is evaluated through a set of numerical results about the static response of sandwich beams with composite multilayered faces subjected to different boundary conditions and external loads; a comparison with three-dimensional exact elasticity solutions (when available) and high-fidelity FE models is also performed. An experimental tests campaign is also carried out to further assess the HZZ model accuracy. It is shown that the proposed approach is valid in determining global and local responses of sandwich beams.

1. Introduction

Composite materials have recently experienced an increasing success and have found a wider and wider range of applicability in different engineering fields (aircraft, space, civil, and naval structures) due to their high stiffness-to-weight and strength-to-weight ratios and to the possibility to be tailored according to the particular application. Sandwich structures exhibit further promising properties as high-energy absorption, impact resistance, noise and vibration reduction. Sandwich panels are commonly constituted by one or more metallic or multilayered composite faces, some adhesive layers and one or more cores [1]; the three main core geometries are honeycomb, corrugated and cellular [2]. Recently, functionally-graded materials and metallic or non-metallic foam cores are increasingly employed to improve the energy dissipation capability of the structure and to enhance the acoustical insulation properties [3]. For some modern civilian and military aircrafts relatively thick laminated composites are adopted in the primary load-bearing structures. This exacerbates some of the typical behaviours of multilayered composite and sandwich laminates as transverse shear and normal deformability and through-the-thickness “zigzag” distribution of in-plane displacements, strains, and stresses [4].

Different approaches may be adopted to face these modelling challenges. Among the structural models available for the analysis of composite and sandwich structure we recall: (i) those having only displacements as primary unknowns and (ii) the so-called mixed models with displacements and stresses (mainly transverse ones) as unknowns [5]. Another classification is based on the assumed distribution along the thickness for the primary unknowns: (a) Equivalent Single Layer (ESL) theories (the unknowns have a priori assumed distributions along the whole thickness of the laminate), (b) Layerwise (LW) theories (the distribution of the unknowns is assumed layer per layer). Among the ESL models based on displacements we recall classical lamination theories (FSDT [6], HSDT [7]). The main advantage of these approaches is their low computational cost and their accuracy for thin and not excessively heterogeneous laminates. LW mixed models are much more accurate and are able to capture the behaviour of thick laminates with large layerwise variations of the engineering constants; their main drawback is the computational complexity, also due to the fact that the number of unknowns depend on the number of layers. From this point of view, LW theories based on displacements are interesting; for some of them the number of unknowns increases with the number of layers (Lu and Liu [8]), for some others (zigzag models by Di Sciuva [9,10]) the number of degrees of freedom does not change and is equal to that of ESL classical theories. These latter theories are able to take into account the zigzag effect and the transverse shear deformability giving through-the-thickness continuous distributions of the relative stresses, thus being a good compromise between computational simplicity and accuracy. A recent enhancement of the classical zigzag theories is the so-called Hermitian ZigZag theory [4,11]; also the transverse normal deformability is considered and the use of the sub-laminate approach leads to very detailed quasi-3D static and dynamic analyses of composite and sandwich structures [12,13].

For a preliminary but accurate design of sandwich structures, detailed models (mainly based on a full reconstruction of the core geometry by shell or solid elements [1,2]) are complex and computationally too expensive. It is then advisable the use of 2D models

* Corresponding author.
E-mail address: marco.degiovanni@polito.it (M. Degiovanni).
URL: http://www.aesdo.polito.it

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based on the classical or higher-order theories quoted in the previous paragraph and where the core is substituted by an equivalent simplified orthotropic layer. A great deal of works is devoted to the estimation of the equivalent engineering constants for honeycomb cores [14], from the classical papers of the '50 about the transverse shear moduli [15,16] to the recent theories able to cover the whole set of engineering constants [17].

Aim of this work is to assess the Hermitian ZigZag theory for the analysis of sandwich structures. In particular, the HZZ beam finite element and the related sub-laminates approach (the core layer being substituted by an equivalent orthotropic material) will be used to determine the displacement and stress fields of sandwich beams subjected to different boundary conditions and external loads; the effect of the facings-to-core relative thickness will also be investigated. Comparison with a classical shear deformation theory and with exact three-dimensional elasticity solutions or high-fidelity 3D FEM analyses will demonstrate the accuracy and computational relative low cost of the proposed HZZ FE model. A further experimental assessment of the approach will be presented where the surface axial strains measured on top and bottom faces of sandwich beams in cylindrical bending will be compared with those obtained via the HZZ FEM approach.

2. The Hermitian ZigZag theory and beam finite element

The Hermitian ZigZag (HZZ) theory for plates has been proposed in [11]; the beam finite element together with the adoption of the sub-laminates approach have been discussed in [18]. More recently, the plate finite element and some applications to the analysis of undamaged and damaged composite plates have been presented [4,19]. In this paragraph we briefly resume the basic properties of the HZZ theory for beams, we describe the related beam finite element, and we discuss the merits of the sub-laminates approach when used for the analysis of sandwich structures.

Let us consider a beam made of N linearly elastic orthotropic layers; its dimensions are L, b, and h with respect to the three axes of a Cartesian coordinate system x_i (x_1 along the beam axis and x_3 in the thickness direction), respectively. The beam HZZ displacement field (V_1, V_3) in the (x_1, x_3) plane may be written as [18]

$$\begin{aligned} V_1(x_1,x_3) &= H^1(x_3)u^0(x_1) - H^2(x_3)w^0_{,1}(x_1) + H^3(x_3)u^h(x_1) \\ &\quad - H^4(x_3)w^h_{,1}(x_1) + \Gamma^0(x_3)\tau^0(x_1) + \Gamma^h(x_3)\tau^h(x_1) \\ V_3(x_1,x_3) &= L^1(x_3)w^0(x_1) + L^2(x_3)w^h(x_1) \end{aligned} \tag{1}$$

where L^i are the through-the-thickness linear Lagrangian polynomials, H^j are the anisotropic cubic Hermitian polynomials [20] (a generalization of the well-known cubic Hermite polynomials for heterogeneous domains), and Γ^k are piecewise cubic polynomials whose value and derivative with respect to x_3 both vanish on top and bottom laminate faces [20]. The L^i, H^j, and Γ^k through-the-thickness shape functions are responsible for a linear variation of the transverse displacement and for a cubic distribution of the axial displacement. Moreover, they guarantee the satisfaction of the transverse shear stress continuity and of traction equilibrium on the external surfaces. Finally, they allow the HZZ theory to adopt, as kinematic unknowns, the axial and transverse displacements and the transverse shear stress of the bottom (u^0, w^0, τ^0) and top (u^h, w^h, τ^h) face of the laminate. The problem of the beam in cylindrical bending in the (x_1, x_3) plane is here considered, thus the required strain field components are ε_{11}, ε_{33}, and γ_{13} and are derived from the displacement field (1) via the usual linear relations

$$\begin{aligned} \varepsilon_{11} &= V_{1,1} \\ \varepsilon^u_{33} &= V_{3,3} \\ \gamma_{13} &= V_{1,3} + V_{3,1} \end{aligned} \tag{2}$$

where the superscript "u" accompanying the transverse normal strain is needed to distinguish it from the same strain component obtained from the constitutive equations written in mixed form [18] and denoted by the "σ" superscript

$$\begin{aligned} \sigma_{11} &= Q_{11}\varepsilon_{11} + S_{33}R_{11}\sigma_{33} \\ \varepsilon^\sigma_{33} &= -S_{33}R_{11}\varepsilon_{11} + S_{33}\sigma_{33} \\ \tau_{13} &= Q_{44}\gamma_{13} \end{aligned} \tag{3}$$

where the Q_{11}, Q_{44}, R_{11}, and S_{33} coefficients are defined in [18]. The transverse normal deformability is taken into account by the HZZ theory: the transverse normal stress σ_{33} is supposed to be through-the-thickness constant and its value is determined in such a way that there is a least squares compatibility between the two strains ε^u_{33} and ε^σ_{33}

$$\int_0^h p(\varepsilon^\sigma_{33} - \varepsilon^u_{33})dx_3 = 0 \tag{4}$$

where p is a weight function [18].

The kinematic unknowns of the HZZ theory are "located" at the top and bottom face of the laminate. The same happens for the degrees of freedom of the beam finite element [18]; the Hermite cubic polynomials are used to approximate the transverse displacement along the axial coordinate x_1 (a rotational degree of freedom θ is introduced for each corner node) and the Lagrangian parabolic polynomials are used for the axial displacement and for the transverse shear stress (a mid-node has to be added where only the u and τ degrees of freedom are active). The Hermitian beam element has 20 degrees of freedom (Fig. 1).

It is an almost natural consequence of the nodes location and of the dof meaning that the developed finite element may be used to sub-divide a beam not only along its axis (classical FEM mesh for 1D problems) but also through its thickness. The obtained thickness sub-domains are called sub-laminates: a sub-laminate is a group of adjacent layers belonging to the whole laminate. We must not think to layers only in the classical "physical" sense, but also in a more "mathematical" one; if needed, a "physical" layer may be divided into two or more "mathematical" layers with the same mechanical properties. As a consequence, a sub-laminate may correspond to the whole laminate thickness, to a group of physical layers or to a group of mathematical ones. For example, when analysing sandwich beams, the use of more than one sub-laminates along the thickness of the core may be useful in order to capture with a higher accuracy the through-the-thickness distribution of such quantities as the transverse displacement and stresses (see next section on numerical results).

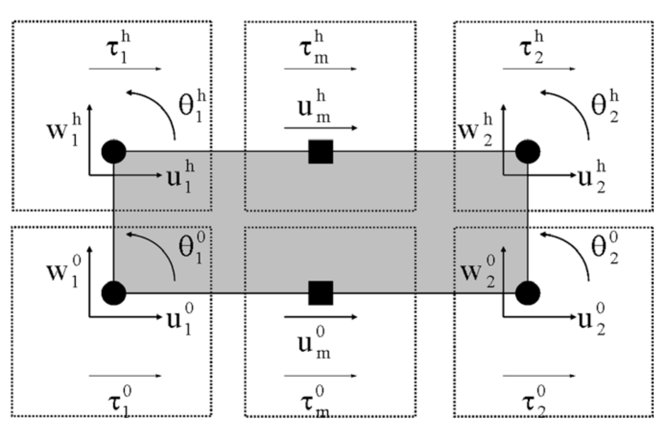

Fig. 1. HZZ beam finite element.