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Three-dimensional asymptotic antiplane shear stress fields at the front of interfacial crack/anticrack type discontinuities in trimaterial bonded plates

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ABSTRACT

An eigenfuntion expansion method is employed for obtaining three-dimensional asymptotic displacement and stress fields in the vicinity of the front of a crack/anticrack type discontinuity weakening/reinforcing an infinite pie-shaped trimaterial plate, of finite thickness, formed as a result of bimaterial (matrix/semiconductor/ARC plus reaction product/scatterer) deposit over a substrate (fiber/ARC/semiconductor). The wedge is of general (unsymmetric) geometrical configuration, and is subjected to antiplane shear (mode III) far field loading. Each material is isotropic and elastic, but with different material properties. The material 2 or the substrate is always taken to be a half-space, while the wedge aperture angle of the material 1 is varied to represent varying composition of the bimaterial deposit. Numerical results pertaining to the variation of the mode III eigenvalues (or stress singularities) with various wedge aperture angles of the material 1 (reaction product/scatterer), are also presented. Hitherto generally unavailable results, pertaining to the through-thickness variations of stress intensity factors for symmetric parabolic load and its skew-symmetric counterpart that also satisfy the boundary conditions on the top and bottom surfaces of the trimaterial plates under investigation, form also an important part of the present investigation.

1. Introduction

Trimaterial bonded wedges are common occurrences in many modern advanced technological applications, such as thin-film solar cells and metal–matrix composites used in aircraft engines. Because of its natural abundance, non-toxicity, and established industry, silicon-based thin-film solar cell technology has engendered considerable promise for GW- and TW-scale energy production, while reducing the bulk material costs of photovoltaic devices [1]. These advantages notwithstanding, silicon suffers from long optical absorption lengths at the red and near-infrared wavelengths, thus necessitating thicknesses of a few hundred microns in order to fully utilize the available power in the incident solar spectrum [1]. In order to mitigate this problem thereby improving the efficiency of thin-film solar cells, Nagel and Scarpulla [1] have recommended scattering from dielectric particles, such as SiO_2. These particles are embedded directly within the semiconductor absorber material with sizes on the order of one wavelength. Importantly, they have ensured that this geometry be fully compatible with the use of an anti-reflective coating (ARC) to maximize light capture, which results in a trimaterial bonded wedge; see the inset of their Fig. 1d.

In metal–matrix composites, such trimaterial bonded wedges result from the presence of reaction products, which are often brittle ceramics or intermetallic compounds [2]. Interfacial chemical reactions are particularly severe during the fabrication stage (at least about 850 °C) when titanium is reinforced by such chemical vapor deposition (CVD) monofilaments as boron or silicon carbide [3]. Experimental results of Warrier and Lin [4] have revealed dissolution of the fibers in the titanium alloy occurring within seconds without forming a continuous layer of reaction products at the interface. Although SiC has a higher refractoriness and chemical inertia than boron, and the SiC CVD-monofilaments retain a much higher fraction of their room temperature mechanical properties up to about 800–1000 °C, SiC monofilament reinforced titanium matrix composites are, according to Martineau et al. [3], regarded as non-equilibrium systems when they are heated at medium or high temperatures. Thermodynamic analysis by Warrier and Lin [4] has further indicated that SiC is unstable in the presence of liquid titanium, dissociating into Si and C, and forming TiC, when the carbon solubility limit in Ti is exceeded.

Although asymptotic behavior of two-dimensional stress fields at the tips of homogeneous/bimaterial and trimaterial wedges (interfacial cracks/anticracks, bimaterial free edges, etc. being special cases) made of isotropic materials and subjected to far-field

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antiplane shear loading, has received considerable and some attention, respectively, in the literature (see e.g., [5–9]), the studies of three-dimensional stress fields in homogeneous/bimaterial wedges are substantially fewer [10–17], while those relating to trimaterial wedges are conspicuous by their total absence. The primary objective of the present investigation is to bridge this important gap. Needless to state, the mathematical difficulties posed by the three-dimensional trimaterial pie-shaped wedge problems are substantially greater than their two-dimensional counterparts (to start with the governing PDE's are much more complicated).

It may also be noted here that the afore-cited three-dimensional stress singularity problems [10–17] have been solved by introducing a novel eigenfunction expansion technique, which has recently been extended to cubic and orthorhombic/orthotropic materials [18–20]. The same technique has also been utilized to compute the asymptotic stress fields in the vicinity of fronts of penny shaped cracks/anticracks weakening homogeneous and bimaterial media [21,22], the penny shaped bimaterial interface crack solution being found to be in agreement with its counterpart due to Willis [23] among others, thus establishing conceptual similarity of this class of problems with their through-thickness counterparts. In addition, the three-dimensional singular stress fields near a partially debonded cylindrical rigid fiber [24], and in the vicinity of the circumferential tip of a fiber-matrix interfacial debond [25,26] have also been derived using the same afore-mentioned three-dimensional eigenfunction expansion technique. Finally, the asymptotic solutions pertaining to the stress fields in the neighborhoods of holes [27], elastic inclusions [28] and bimaterial holes [29] have also been shown to be in agreement with their counterparts derived by Folias [30–32], using Lure's symbolic method. These facts not only lend credence to the validity of the afore-mentioned three-dimensional eigenfunction expansion approach, but also reinforce the afore-mentioned conceptual as well as mathematical similarity of and linkages among the afore-cited classes of three-dimensional stress singularity problems.

In what follows, the above-mentioned eigenfuntion expansion method is employed for obtaining three-dimensional asymptotic displacement and stress fields in the vicinity of the front of a crack/anticrack type discontinuity weakening/reinforcing an infinite pie-shaped trimaterial wedge, of finite thickness. The wedge is of general (unsymmetric) geometrical configuration, and is subjected to antiplane shear (mode III) far field loading. Each material is isotropic and elastic, but with different material properties. The material 2 or the substrate is always taken to be a half-space, while the wedge aperture angle of the material 1 is varied to represent varying composition of the bimaterial deposit. Numerical results pertaining to the variation of the mode III eigenvalues (or stress singularities) with shear modulus ratio, as well as with various wedge aperture angles of the material 1 (reaction product/scatterer), are also presented.

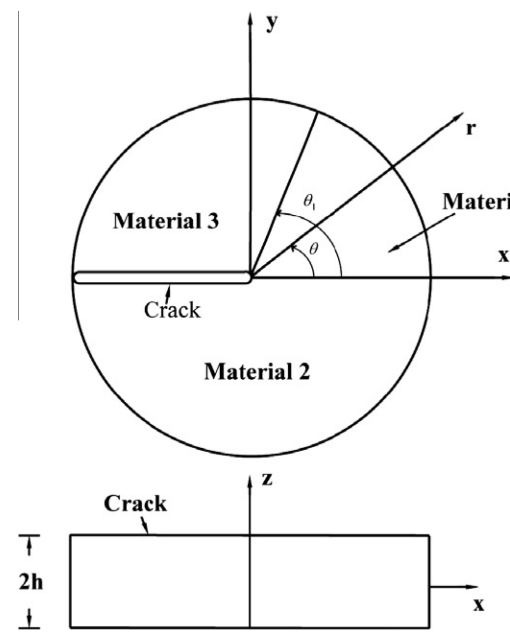

Fig. 1. Schematic of a trimaterial plate with a bimaterial interfacial crack.

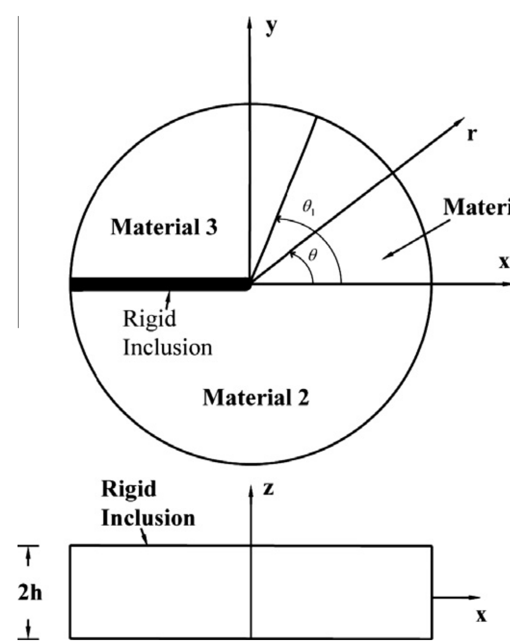

Fig. 2. Schematic of a trimaterial plate with a bimaterial interfacial anticrack (very thin lamellar rigid inclusion).

2. Statement of the problem

The cylindrical polar coordinate system, (r,θ,z), is convenient to describe the deformation behavior in the vicinity of the front of an interfacial crack/anticrack weakening/reinforcing an infinite pie-shaped trimaterial plate, of thickness $2h$ (Figs. 1 and 2). The material 2 (or substrate) is always taken to be a half-space, while the wedge aperture angle of the material 1 is assumed to be θ_1 at the crack/anticrack front. The wedge aperture angle of material 3 at the crack/anticrack front is $\pi - \theta_1$. Here, the z-axis ($z \geqslant [-h, h]$) is placed along the straight crack/anticrack front, while the coordinates r, θ, are used to define the position of an element in the plane of the plate (see Figs. 1 and 2). The bimaterial interface between materials 1 and 2 is located at $\theta = 0$, i.e., it coincides with the positive x-axis, while the remaining two bimaterial interfaces are located at $\theta = \theta_1$, π (Figs. 1 and 2). In what follows, a through-thickness crack/anticrack (very thin rigid inclusion) is introduced at the bimaterial interface located at $\theta = \pi$ (Figs. 1 and 2). Components of the displacement vector in the radial and tangential directions are represented by U_r, U_θ, while the component in the z-direction is denoted by U_z. In the absence of body forces, the coupled partial differential equations in terms of the displacement functions U_r, U_θ, and U_z are given as follows: