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Reducing vibration of sandwich structures using antiresonance frequencies

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ABSTRACT

A method for reducing vibration of a sandwich structure using the antiresonance technique is presented. It is found that with appropriate resonators, the motion of the sandwich structure due to disturbances with certain frequencies can be effectively suppressed. A simple two-degree-of-freedom system consisting of an absorbing mass connected by springs to a drive mass is introduced and used to interpret the vibration behavior of the sandwich structure with resonators.

1. Introduction

Sandwich structures widely used in aircraft, marine, and wind turbine blade structures which are subjected to periodic dynamic loads. In order to avoid the consequences of divergent motion, passive solutions including mass balancing and structural modification or active control techniques have been employed [1,2]. Also, a few resonators have investigated the optimization of the geometric and material properties of sandwich structures for resisting acoustic disturbances [3,4]. Moreover, the topology design of periodic multifunctional core structures and the relationship between the topology and performance has been presented and utilized for sound/vibration reduction [5,6]. Recently, Chen et al. [7] proposed the idea of embedding resonators in the core of the sandwich structure and show the dynamic disturbances can be attenuated as they propagate along the sandwich beam.

In this study, the resonators with proper resonance frequencies are proposed for suppressing the resonance of the sandwich structure. The whole system operates on the same principle as the dynamic vibration absorber. This novel idea arises from a two-degree-of-freedom system which is composed of an absorber mass connected by linear springs to a drive mass subjected to a harmonic exciting force [8]. It is found that if the driving frequency is close to the antiresonance frequency of the whole system, the absorber mass will generate an equal force but opposite to the external or the drive force. As a result, the original oscillating system remains stationary since the absorber mass exactly cancels the effects of the input force.

Applying this concept to solid structures offers an additional possibility of suppressing the resonance of structures. The sandwich structure acts more or less like the drive mass while the internal mass of the resonator as the absorber mass. The steady-state and transient responses of a sandwich beam with appropriate internal resonators are studied. It is expected that disturbances dominated by frequencies that are close to the antiresonance frequency of the whole structure cannot excite the sandwich beam into transverse motion and the beam would appear almost motionless.

2. Force vibration of two-degree-of-freedom system

A diagram of a two-degree-of-freedom system is shown in Fig. 1 in which m_1 is connected through a spring (k_1) to the fixed end, and m_2 is connected through a spring (k_2) to m_1. A harmonic forcing function acts on mass m_1. m_1 and m_2 are usually referred to as the drive mass and the absorber mass, respectively; k_1 and k_2 as the drive mass spring and the absorber mass spring. The equations of motion can be readily expressed as

$$m_1\ddot{u}_1 + k_1u_1 + k_2(u_1 - u_2) = \overline{F}\sin\omega t \tag{1}$$

$$m_2\ddot{u}_2 + k_2(u_2 - u_1) = 0 \tag{2}$$

where $\overline{F}$ is the force amplitude, u_1 and u_2 are the displacements of mass m_1 and m_2, respectively.

The solution for the steady-state harmonic motion can be obtained by substituting the displacements

$$u_i = \bar{u}_i\sin\omega t \quad (i = 1, 2) \tag{3}$$

in Eqs. (1) and (2) with the results of the force–displacement ($\overline{F} - \bar{u}_1$) relation and the displacement relation of two masses, namely

$$\left[-m_1 - \frac{(k_1 + k_2)m_2}{k_2 - \omega^2 m_2}\right]\omega^2\bar{u}_1 + \left[\frac{k_1k_2}{k_2 - \omega^2 m_2}\right]\bar{u}_1 = \overline{F} \tag{4}$$

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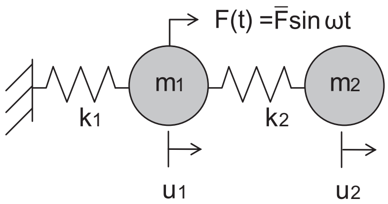

Fig. 1. Two degrees of freedom system.

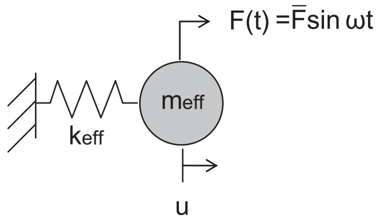

Fig. 2. The equivalent model.

$$\bar{u}_2 = \frac{k_2}{k_2 - m_2\omega^2}\bar{u}_1 \tag{5}$$

By solving Eqs. (4) and (5), the displacements $\bar{u}_1$ and $\bar{u}_2$ can be obtained as

$$\bar{u}_1 = \frac{\bar{F}}{k_1}\frac{\left(1 - \frac{\omega^2}{\omega_2^2}\right)}{\left[\left(1 + \frac{k_2}{k_1} - \frac{\omega^2}{\omega_1^2}\right)\left(1 - \frac{\omega^2}{\omega_2^2}\right) - \frac{k_2}{k_1}\right]} \tag{6}$$

$$\bar{u}_2 = \frac{\bar{F}}{k_1}\frac{1}{\left[\left(1 + \frac{k_2}{k_1} - \frac{\omega^2}{\omega_1^2}\right)\left(1 - \frac{\omega^2}{\omega_2^2}\right) - \frac{k_2}{k_1}\right]} \tag{7}$$

where ω_1 and ω_2 are the resonant frequencies of the isolated $m_1 - k_1$ and $m_2 - k_2$ systems, respectively, i.e.,

$$\omega_1 = \sqrt{k_1/m_1} \tag{8}$$

$$\omega_2 = \sqrt{k_2/m_2} \tag{9}$$

An antiresonance frequency can be determined from Eqs. (6) and (7). It is apparent that when the numerator of Eq. (6) vanishes and the denominator has a finite value, $\bar{u}_1$ becomes zero and hence ω_{ant} is obtained as

$$\omega_{ant}^2 = \frac{k_2}{m_2} \tag{10}$$

In addition, the system resonance frequencies are acquired by equating the denominators of the system responses to zero. Thus

$$\left(1 + \frac{k_2}{k_1} - \frac{\omega^2}{\omega_1^2}\right)\left(1 - \frac{\omega^2}{\omega_2^2}\right) - \frac{k_2}{k_1} = 0 \tag{11}$$

Eq. (11) is a quadratic equation in ω^2. Hence it is evident that two finite values of ω^2, which satisfy Eq. (11), can be found. In other words, two forcing frequencies would result in the system resonance. Moreover, as mass ratio m_2/m_1 increases, the resonance frequency separation increases.

If this two-degree-of-freedom system is treated as a one-degree-of freedom model, an effective mass must be introduced in order that the representative mass can effectively describe the vibration of m_1. In other words, the identity of the internal mass m_2 would be ignored and its effect would be absorbed by introducing an effective mass m_{eff} as depicted in Fig. 2.

The force–displacement relation for the single mass model is given by

$$-m_{eff}\omega^2\bar{u} + k_{eff}\bar{u} = \bar{F} \tag{12}$$

Comparing Eqs. (4) and (12) we obtain the effective mass and effective spring constant, respectively, as

$$m_{eff} = m_1 + \frac{(k_1 + k_2)m_2}{k_2 - m_2\omega^2} \tag{13}$$

$$k_{eff} = \frac{k_1 k_2}{k_2 - m_2\omega^2} \tag{14}$$

Rewriting Eq. (12) gives the displacement amplitude of m_{eff}, i.e.,

$$\bar{u} = \frac{\bar{F}}{k_{eff} - m_{eff}\omega^2} \tag{15}$$

From Eq. (15), it is evident that at the antiresonance frequency ω_{ant} defined by

$$\omega_{ant}^2 = k_2/m_2 \tag{16}$$

both m_{eff} and k_{eff} become unbounded, and thus, $\bar{u}$ vanishes.

3. Force vibration of a sandwich beam with internal resonators

A supported sandwich beam with internal resonators, subjected to a uniform harmonic pressure excitation of frequency ω, is considered and illustrated in Fig. 3. Each resonator is composed of a mass m_r and two linear springs with a spring constant denoted by k_r. A representative sandwich beam element (a unit cell) and the equivalent model viewed as the combination of an equivalent sandwich beam element attached by a mass with a spring are depicted in Figs. 4a and b, respectively [7]. The sandwich beam is treated as a Timoshenko beam with the equivalent material properties including the bending rigidity EI, shear rigidity GA, mass per unit length ρA, and rotary inertia ρI. The beam mass in a unit cell is denoted by m_b, and $m_b = \rho A \times a$ where a is the spacing of the resonators. Instead of solving complicated differential equations of the beam model, the resonant frequencies and antiresonance frequency of such a sandwich structure can be obtained by using

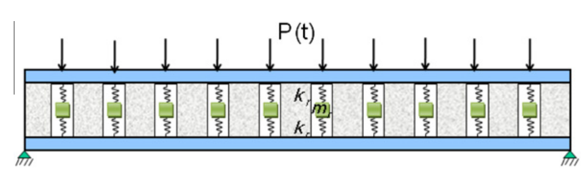

Fig. 3. A supported sandwich beam with resonators under a uniform harmonic pressure.

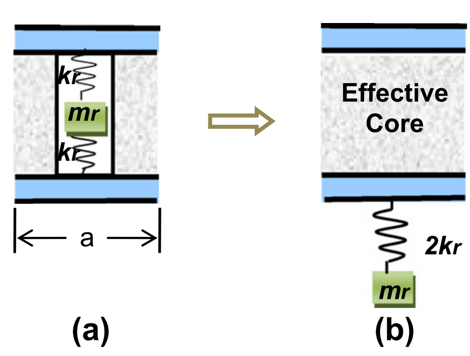

Fig. 4. Schematic diagram of (a) a unit cell and (b) its equivalent model.