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Nonlinear behavior of functionally graded circular plates with various boundary supports under asymmetric thermo-mechanical loading

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ABSTRACT

The equilibrium equations of the first-order nonlinear von Karman theory for FG circular plates under asymmetric transverse loading and heat conduction through the plate thickness are reformulated into those describing the interior and edge-zone problems of the plate. A two parameter perturbation technique, in conjunction with Fourier series method is used to obtain analytical solutions for nonlinear behavior of functionally graded circular plates with various clamped and simply-supported boundary conditions. The material properties are graded through the plate thickness according to a power-law distribution of the volume fraction of the constituents. The results are verified with known results in the literature. The load–deflection curves for different loadings, boundary conditions, and material constant in a solid circular plate are studied and discussed. It is shown that the behavior of FG plates with clamped or simply-supported boundary conditions are completely different. Under thermo-mechanical loading, snap-through buckling behavior is observed in simply-supported FG plates which are immovable in radial direction. Moreover, it is found that linear theory is inadequate for analyzing FG and also homogenous plates with immovable boundary supports in radial direction and subjected to thermal loading, even for deflections that are normally considered small.

1. Introduction

Functionally graded materials (FGMs) are microscopically heterogeneous materials whose composition and therefore properties are designed to vary continuously usually in one direction. They are mostly made from ceramic and metal especially for usage in high temperature environments, the field in which their concept was originally introduced [1]. They had attracted many attentions and their application is growing up, since many problems which exist in laminated composite materials are eliminated in FGMs due to smooth variation of their composition.

Studies on nonlinear behavior of FG plates are, however, rare in comparison with those available on linear analyses of FG rectangular [2–5] and circular [6–8] plates. Reddy [9] developed Navier's solutions for rectangular plates and finite element models to study the nonlinear dynamic response of FG plates using higher-order shear deformation plate theory. Based on the classical nonlinear von Karman plate theory, an analytical solution in terms of Fourier series was obtained by Woo and Meguid [10] for the nonlinear bending of functionally graded plates and shallow shells under transverse mechanical loads and a temperature field. Based on the higher-order shear deformation plate theory with the von Karman nonlinearity, Woo et al. [11] provided an analytical solution for the post buckling behavior of functionally graded plates and shallow shells under compressive edge loads and a temperature field. Na and Kim investigated nonlinear bending [12] and thermal buckling and post buckling [13] of FG clamped square plates based on the Green–Lagrange nonlinear strain–displacement relation using a 3-D finite element method. Thermo-mechanical loading is considered in [12]. Yang and Shen [14] investigated the large deflection and post buckling responses of functionally graded rectangular plates with clamped supports on two opposite edges under transverse and in-plane loads using the classical von Karman plate theory. Based on Reddy's higher order shear deformation plate theory with von Karman-type of kinematic nonlinearity, Shen presented nonlinear thermal bending [15] and post buckling [16] analyses for a simply-supported FG plate using a two step perturbation technique. Heat conduction is considered in the thickness direction. Based on first-order shear deformation plate theory (FSDT) with the von Karman nonlinearity, Zhao and his co-workers presented bending [17] and post buckling [18] analyses of functionally graded plates under edge compression and transverse heat conduction using the element-free *kp*-Ritz method. Prakash et al. [19] investigated nonlinear behavior of FG plates exposed to high temperature on the ceramic surface using

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neutral surface-based FSDT with the von Karman nonlinearity. They reported snapping phenomenon in FG rectangular and skew plates under thermal loading. Based on higher-order shear deformation plate theory and full definition of Green strain tensor, Han et al. [20] presented finite element solutions for nonlinear behavior of laminated composite and sigmoid functionally graded anisotropic plates and shells. They reported snap-through buckling in simply-supported FG cylindrical shells under transverse mechanical loading.

From the literature review, it is evident that analysis of nonlinear bending and post buckling of FG circular plates [21–30] are much less than rectangular plates [9–20], while circular plates made of functionally graded materials are often employed as a part of engineering structures. Gunes and Reddy [21] carried out geometrically nonlinear analysis of FG circular plates with different boundary conditions subjected to mechanical and thermal loads using all the terms in Green–Lagrange strain tensor. Based on FSDT with nonlinear von karman assumptions, Sepahi et al. [22] investigated axisymmetric large deflections of thermo- mechanical loaded annular FGM plates on nonlinear elastic foundation using differential quadrature method. Based on the classical nonlinear von Karman plate theory, the axisymmetric bending and post buckling of functionally graded thin circular plates subjected to mechanical and thermal loadings were studied by Ma and Wang [23]. Li et al. [24] studied nonlinear thermo-mechanical post buckling of an imperfect and perfect clamped FG circular plate using shooting method. Several authors have investigated the buckling of functionally graded circular plates [25,26] in which the existence of bifurcation buckling for both clamped and simply-supported FG plates is assumed and the buckling point is searched using an eigen-value analysis. However, there is the discussion that plates which are unsymmetric with respect to their midplane (i.e., unsymmetrically laminated plates and FG plates) will typically bend when subjected to in-plane compressive loads and bifurcation buckling only happens under certain conditions [27]. Singh et al. [28] showed that flatness of the plate during the prebuckling regime guarantee bifurcation buckling but it is not essential for the bifurcation instability phenomenon to occur. They reported a few plate configurations for which bending curvatures do occur during the prebuckling stage but nevertheless still exhibit bifurcation instability. For FG plates, it is shown that due to edge compression, bifurcation-type buckling happens in fully clamped FG plates, while bending occurs in simply-supported FG plates [11,13,16,23,24].

Prakash and Ganapathi [29] investigated asymmetric free vibration and thermoelastic stability of FG circular plates using finite element procedure. Based on FSDT, Nosier and Fallah presented analytical solution for asymmetric linear [7] and nonlinear [30] bending of FG circular plates with various clamped and simply-supported boundary conditions. In the linear analysis [7], thermo-mechanical loading is considered while only mechanical loading is considered in the nonlinear analysis [30] in which a one parameter perturbation technique is used with middle point transverse deflection of the plate as the perturbation parameter. Here, asymmetric nonlinear bending and post buckling behavior of FG plates subjected to both thermal and thermo-mechanical loadings are investigated using a two parameter perturbation technique. From the review of literature it appears that not only very little work is available on asymmetric bending and post buckling of FG circular plates [7,29,30], but also analysis of axisymmetric bending [21–23] and post buckling [23,24] of FG circular plates have been taken up by a few researchers.

In the present study, analytical solution is presented for the nonlinear equilibrium equations of FSDT governing the nonlinear behavior of FG circular plates subjected to asymmetric transverse loading and a temperature variation through the plate thickness. Geometric nonlinearity in the von Karman sense is considered. Here, based on the method developed in [30] which uses two potential functions (i.e., boundary layer and stress functions), the equilibrium equations which are five nonlinear coupled equations due to the bending-extension coupling in FG materials, are reformulated into three equations, for which an analytical solution may be presented for different types of clamped and simply-supported boundary conditions. To this end, a standard two parameter perturbation technique with different perturbation parameters is adapted. The effects of nonlinearity, material properties, and boundary conditions on nonlinear bending and post buckling behavior of a solid circular plate are studied and discussed in detail.

2. Theoretical formulation

A functionally graded circular plate of inner and outer radii of, respectively, a and b and thickness h is considered here. The geometry of the plate and the coordinate system are shown in Fig. 1. Here, FGMs are modeled as a nonhomogenous isotropic linear thermoelastic material whose properties, $\mathcal{P}$, vary continuously through the plate thickness, as a function of the volume fraction and properties of the constituent materials. Assuming the plate is made from a mixture of ceramic and metal, $\mathcal{P}$ can be expressed as ([6,23,31]):

$$\mathcal{P}(z) = (\mathcal{P}_m - \mathcal{P}_c)\left(\frac{h-2z}{2h}\right)^n + \mathcal{P}_c \tag{1}$$

where subscripts c and m refer to ceramic and metal, respectively and n is the power-law index that takes values greater than or equal to zero. Eq. (1) is obtained using the linear rule of mixture which is the simplest estimate of the effective material properties at a point in a dual-phase metal-ceramic material. A survey on different models to ascertain effective properties of an FGM is included in [2]. In the present study, relation (1) will be used as a model for Young's modulus E, thermal conductivity $\overline{K}$, thermal expansion coefficient α and Poisson's ratio υ of FG plates.

2.1. Thermal analysis

In thermal analysis it is assumed that the temperature variation is only in the thickness direction and constant surface temperatures at the ceramic and metal rich surfaces are imposed. The one-dimensional steady state heat conduction equation in the z-direction is given by:

$$-\frac{\mathrm{d}}{\mathrm{d}z}\left(\overline{K}(z)\frac{\mathrm{d}T(z)}{\mathrm{d}z}\right) = 0 \tag{2}$$

with the boundary condition $T(h/2) = T_c$ and $T(-h/2) = T_m$. Here, a stress-free state is assumed to exist at $T_0 = 25$ °C. The thermal conductivity coefficient $\overline{K}(z)$ is assumed here to obey the power-law relation in (1). Eq. (2) is solved analytically for $n = 0$, 0.5 and integer

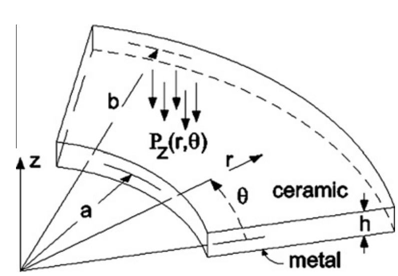

Fig. 1. Geometry of FG circular plate and coordinate system.