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Coupled thermo-mechanical analysis of one-layered and multilayered plates

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ABSTRACT

The paper considers a fully coupled thermo-mechanical analysis of one-layered and multilayered isotropic and composite plates. In the proposed analysis, the temperature is considered a primary variable as the displacement; it is therefore directly obtained from the model and this feature permits the temperature field to be evaluated through the thickness direction in three different cases: – static analysis with imposed temperature on the external surfaces; – static analysis of structures subjected to a mechanical load, with the possibility of considering the temperature field effects; – a free vibration problem, with the evaluation of the temperature field effects. In the first case, imposing a temperature at the top and bottom of the plate, the static response is given in term of displacements, stresses and temperature field; the proposed method is very promising if compared to a partially coupled thermo-mechanical analysis, where the temperature is only considered as an external load, and the temperature profile must be a priori defined: considering it linear through the thickness direction or calculating it by solving the Fourier heat conduction equation. In the second case, a mechanical load is applied. The fully coupled thermo-mechanical analysis gives smaller displacement values than those obtained with the pure mechanical analysis; the temperature effect is not considered in this latter approach. The third case is the free vibration problem. The fully coupled thermo-mechanical analysis permits the effect of the temperature field to be evaluated: larger frequencies are obtained with respect to the pure mechanical analysis. Carrera's Unified Formulation is applied to obtain several refined models with orders of expansion in the thickness direction, from linear to fourth-order, for displacements and temperature. Both equivalent single layer and layer wise approaches are considered for the multilayered plates. At present, no benchmarks are available within the framework of a fully coupled theory. This work aims to fill this gap.

1. Introduction

Thermoelasticity is a branch of applied mechanics that is concerned with the effects of heat on the deformation and stresses of solid bodies, which are considered to be elastic. It is therefore considered as an extension of the conventional theory of isothermal elasticity to those processes in which deformation and stresses are produced not only by mechanical forces, but also by temperature variations. Thermoelastic processes are not totally reversible: the elastic part may be reversed (the deformations caused by heat are theoretically recoverable through cooling), but the thermal part may not be reversed because of the dissipation of energy that takes place during heat transfer. The effect of the temperature field on the deformation field is not a one-way phenomenon; it is in fact well known that a deformation of the body produces changes in its temperature. These features demonstrate that the mechanical and thermal aspects are coupled and inseparable [1]. Therefore, this coupling considerably complicates the computational aspect of solving actual thermoelastic problems. As suggested in the introduction to Nowinski's book [1], it is possible to say: *"Practically speaking, it is generally possible to discount the coupling and to evaluate the temperature and deformation fields, in this order, separately"*. The thermoelastic problem, where the temperature and deformation fields are discounted, is here defined as a partially coupled thermo-mechanical problem.

Partially coupled thermo-mechanical models are extensively employed in the analysis of typical aeronautical structures, such as one-layered isotropic and multilayered composite plates and shells, where the temperature variations are one of the most important factors for the stress fields that can cause failure of the structures [2–4]. These structures are subject to severe thermal environments, such as high temperatures, high gradients and cyclic changes in temperature. Because of these implications, the effects of both high-temperature and mechanical loadings have to be considered in the design process. An accurate description of local stress fields in the layers becomes mandatory to prevent thermally loaded structure failure mechanisms. Computational models,

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developed to study the behavior of high-temperature composite plates and shells, make use of partially coupled thermo-mechanical analysis [4]. The temperature is only considered as an external load and the temperature profile must be a priori defined: considering it "a priori" through the thickness direction [5–10] or calculating it by solving the Fourier heat conduction equation [11–15].

Wu and Chen [5] have described displacements and stresses in laminated structures under thermal bending, assuming a linear temperature profile through the thickness direction. In [6], Brischetto and Carrera have used the same linear temperature profile to asses the accuracy of several refined models for the static analysis of multilayered composite shells. A linear temperature profile through the thickness direction has also been considered by Bhaskar et al. [7] and Khare et al. [8] to analyze the deflection of composite laminates and laminated or sandwich shells, respectively. Khdeir [9] has solved the thermoelastic governing equations by assuming a linear or constant temperature profile through the thickness. An interesting method to analyze the thermal stresses in shells is the use of Cosserat surfaces, as done by Birsan [10] for two given temperature fields.

Other models, given in the open literature, calculate the temperature profile through the thickness: sometimes, "a priori" assumption can lead to very large errors. In the case of multilayered anisotropic structures, the temperature profile is never linear, even when the plate or shell is thin: an incorrect temperature profile gives an erroneous thermal load which leads to larger errors, even though the structural model is accurate [11,12]. A finite element shell has been developed by Rolfes et al. [13] to analyze composite structures simultaneously loaded by mechanical and thermal loads; the temperature profile has been presumed linear or quadratic in the thickness direction and then introduced into the Fourier heat conduction equation. The Fourier heat conduction equation has been solved for multilayered composite shells and for functionally graded material plates in [14,15], respectively. The calculated temperature profile gives an appropriate thermal load to correctly investigate the thermal deflection of such structures.

The present work proposes a fully coupled thermoelastic analysis where both temperature and displacement fields are primary variables in the thermo-mechanical governing equations. This fully coupled model permits several problems, which are of particular interest in the aeronautics and space fields, but not only, to be analyzed, in a very efficient and simple way. Therefore, the case of structures with imposed temperature on the external surfaces is easily solved without the need to a priori define the temperature profile in the thickness direction. The temperature is a primary variable of the problem, and the values of temperature at the top and bottom are directly imposed: the fully coupled thermo-mechanical governing equations directly give the displacements and the temperature through the thickness direction. In order to calculate the displacements, the partially coupled governing equations instead need an a priori temperature profile in the thickness direction (assumed [5–10] or calculated [11–15]) to define the thermal load. The other two possible applications of the fully-coupled governing equations are: an external applied mechanical load and the free vibration problem. These two cases are also investigated in this paper, but a relevant simplification is made: the variation in time of the temperature is not considered, and this means that these two problems are investigated at equilibrium conditions. This great simplification will be removed in a future work in order to investigate the evolution in time of temperature, strain and stress in such problems.

In the open literature, a small amount of work has been devoted to the coupled thermo-mechanical analysis of structures (both thermoelastic and thermoplastic analysis), and only few of them give numerical results. Altay and Dökmeci [16] have described the physical behavior of thermoelastic continuum by means of opportune variational principles. The stress equations of motion and the equation of heat conduction have been written as divergence equations. The strain-mechanical displacement relations and Fourier's heat conduction law have been written as gradient equations. The same authors have extended this method to thermopiezoelectric mediums in [17] by simply adding the charge equation of electrostatics in the divergence equations and the electric field-electric potential relations in the gradient equations. Das et al. [18] have avoided the use of the thermoelastic potential to solve the general problem of one-dimensional linearized simultaneous equations of thermoelasticity. Displacement and thermal fields have been obtained in the Laplace transformation domain. This method could be very useful in thermoelasticity or other coupled fields.

Some papers have investigated the thermo-mechanical coupling when a temperature is applied to the structure. Cannarozzi and Ubertini [19] have proposed a variational method for linear coupled quasi-static thermoelastic analysis. The variational support is a statement in terms of displacement, temperature, stress and heat flux. The statement has been based on the hybrid stress formulation for the elastic part and on the mixed flux-temperature formulation for the thermal part, and it has included the rate dependent terms of the energy balance equations and the initial conditions. Thermal balance and initial conditions have been weakly enforced using temperature as a Lagrange multiplier, and the thermoelastic dissipation term has been expressed via the constitutive equations, in terms of stress and temperature rates. The local displacement, temperature, stress and heat flux errors have been measured in time when temperature and/or displacement have been applied. Comparisons between coupled and uncoupled analysis, and the accuracy and efficiency of the coupled theory have been demonstrated in [20]. A higher-order zig-zag plate theory (for an exhaustive overview on zig-zag models see [21]) has been developed to refine the prediction of the fully coupled mechanical, thermal, and electric behavior. Both in-plane displacement and temperature fields, through the thickness have been constructed by superimposing a linear zig-zag field on to the smooth, globally cubic varying field. The given theory is suitable for the predictions of fully coupled behavior of thick, smart composite plates under combined mechanical, thermal, and electric loads. The same authors have extended the proposed analysis to a three-node triangular finite element in [22]. Ibrahimbegovic et al. [23] have presented a thermo-mechanical coupling model for folded plates or non-smooth shells which can be used for the analysis of the fire-resistance of cellular structures. Thermo-mechanical coupling has been considered, including radiative exchanges and an operator split solution procedure with different time steps. The motivation for the work was the development of predictive models that would be capable of describing the inelastic behavior of cellular structures, build either of folded plates and/or non-smooth shells, under sustained long term high-temperature effects. In [24], Lee has fully discussed the thermoelasticity problem of multilayered adiabatic and clamped hollow cylinders whose boundaries are subjected to time-dependent temperatures. Solutions for the temperature, displacement and thermal stress distributions have been obtained in both a transient and steady state. The method has developed stable solutions at a specific time. Tanaka et al. [25] have proposed a new boundary element method for the analysis of quasi-static problems in coupled thermoelasticity. Through some mathematical manipulations of the Navier equation in elasticity, the heat conduction equation has been transformed into a simpler form, similar to the uncoupled-type equation with the modified thermal conductivity which shows the coupling effects. This procedure has made it possible to treat the coupled thermoelastic problem as an uncoupled one.