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# Decomposition method for solving optimal material orientation problems

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ABSTRACT

The strain energy density is considered as a measure of the stiffness/flexibility of the composite structure. A methodology for determining the stationary points of the strain energy density in anisotropic solids is developed. The methodology proposed is based on new problem formulation, derivation and analysis of optimality conditions, and decomposition method. The optimal material orientation problem is formulated in terms of strains. The optimality conditions derived cover different material symmetries, linear and also some non-linear material models. The complexity analysis of the optimality conditions has been performed. The proposed approach allows to divide the solution of the optimal material orientation problem into less complicated subtasks.

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## 1. Introduction

The stiffness/flexibility of the composite structure is affected among size, shape, topology, etc. and by the material orientation. In [1] Banichuk introduced energy based formulation for optimal material orientation problem according to which the elastic energy density is considered as a measure of the stress–strain state. This approach is used most commonly up to now. Pedersen [2,3], Sacci and Rovatti [4] applied this approach for optimal design of 2D linear elastic orthotropic materials. In [2] the closed form analytical solution for optimal material orientation problem of linear elastic 2D orthotropic materials is given including analysis of global and local extremes. The results for non-orthotropic 2D linear elastic materials are given in [5].

In [6–8] different non-linear elastic material models are proposed for solving oriental design problems. In all these papers the effective strain (stress) has been used as a scalar measure of the strain (stress) state. In [6,8] the non-linear material behavior is simulated on the basis of a power law with one term and a series expansion, respectively. In [7] the stress strain relation is described by more general function of effective strain, given in the implicit form.

Orientational design problems of 3D orthotropic materials are studied by Seregin and Troitski [9], Rovati and Taliercio [10,11], Cowin [12], etc. In [9] and [10,11] the potential energy of deformation and the specific elastic energy density are subjected to minimization, respectively. In all these papers it is pointed out, that the stress and strain tensors are coaxial at the optimum. In [9] the optimality conditions in terms of stresses for general orthotropic material are derived and the solution modes are discussed. In [10] the optimization problem is formulated in terms of Euler angles. Collinearity of principal directions of stress and strain at the optimum is derived from the stationary condition of the strain energy density. Complete analytical solution for body with cubic symmetry in terms of strains is given. In both, Rovati and Taliercio [11] and Cowin [12] thoroughgoing analysis of the 3D optimal material orientation problem is performed. In these papers the general non-orthotropic material with 21 independent coefficients is considered and the coaxiality condition of the stress and strain tensors is converted into simplified form using decomposition of the elasticity tensor. The final form of the optimality conditions, given in [11,12] in terms of compliance and constitutive tensors, respectively, is a quite similar, but the interpretation is principally different. In [12] the search for optimal solution is performed for all stress states, which leads to condition that all compliance coefficients (mutually rotated) must vanish. In latter case the closed formed solution of optimality conditions is not too complicated, but the unique solution does not exist for all material symmetries. In [11] a fixed strain (stress) state is considered. In the latter case the solution of optimality conditions is complicated task in the case of general orthotropic or non-orthotropic materials.

An overview of the optimization techniques dealing with composites laminates with uniform stacking sequence through their entire structure is given in [13]. The properties of different optimization methods (including gradient based and direct search methods, specialized algorithms) utilized for composite lay-up design are compared. An approach proposed in [14] for design of composite laminates transforms a bending design problem into simpler extension design problem. Interesting particular design problems – development of optimal biomimetic composite structures are discussed in [15]. This paper is focused on experimental study of optimal composite structures in the forewings of beetles. A novel schematic model of reticular chitin fibers containing the bridging

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fibers between the fibers has been introduced. Such structure has certain advantages in comparison with an ordinary cloth weaving: unbendable, without concentrated stress at the cross points between the fibers.

In the current paper the optimal material orientation problem is formulated in terms of strains considering the strain energy density as a measure of the stress–strain state. The strain components are considered as design variables. Such an approach allows to derive the necessary optimality conditions in terms of strains for different material symmetries and stress–strain relations. The optimality conditions in terms of strains have been derived by authors also in [16,17], but these results are material symmetry specific (the optimization problem has been formulated initially in terms of Euler angles and converted to strains using relations between the strain components and its derivatives with respect to Euler angles). Next the complexity analysis, simplification and solution of the optimality conditions have been performed for specified material symmetries. It is shown that in the case of linear elastic 3D orthotropic materials the order of optimality conditions given in terms of strains can be reduced up to solving one-sixth order algebraic equation for the solution mode where all shear stresses are nonzero (the remaining solution modes are covered with closed form analytical solutions).

An approach proposed above allows to consider different design variables given as functions of strain components. Numerical algorithm based on global optimization technique (hybrid GA) is treated as an alternative solution and the results are compared.

## 2. Theoretical framework

An approach given in the current paper includes new problem formulation, derivation of the optimality conditions and the decomposition method described in detail below. This approach can be utilized for 3D materials with special symmetries, general 3D anisotropic materials (21 independent material parameters), 2D anisotropic materials, linear and non-linear elastic material models.

### 2.1. Problem formulation

In the following the strain energy density is considered as a measure of the stress–strain state

$$U = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} \rightarrow \min(\max), \tag{1}$$

where $\sigma_{ij}$ and $\varepsilon_{ij}$ stand for the components of the stress and strain tensors, respectively. It is assumed that elastic solid is subjected to a fixed strain (stress) field. The goal is to determine the optimal material orientations corresponding to stationary points of the strain energy density. Most commonly, the Euler angles or the components of the rotation tensor are considered as design variables (3D anisotropic elasticity) and the optimization problem is formulated as [10–12]

$$J_{\text{Energy}} = U(\theta_i, C_{ijkl}) \rightarrow \min(\max), \tag{2}$$

$$J_{\text{Energy}} = U(Q_{ij}, C_{ijkl}) \rightarrow \min(\max). \tag{3}$$

In (2), (3) $\theta_i$, $Q_{ij}$ and $C_{ijkl}$ stand for the Euler angles, the components of the rotation tensor and elasticity tensor, respectively.

In the current study the strain components $\varepsilon_{ij}$ are considered as design variables and the optimization problem is formulated as

$$J_{\text{Energy}} = U(\varepsilon_{ij}, C_{ijkl}) \rightarrow \min(\max). \tag{4}$$

The objectives (2)–(4) are given in a similar form, but their implementation is a quite different. First, note that the optimization problem considered is unconstrained in the case of formulation (2) and constrained in the case of formulations (3), (4). The Euler angles (parameters) are included in strain energy density through the rotation tensor, which is applied to the constitutive matrix in [11,12] and to the strain components in [16] and herein. In the latter case the transformation formulas for strains are given as

$$\varepsilon_{ik} = \varepsilon_j Q_{ji} Q_{jk}, \tag{5}$$

where $\varepsilon_j (j = I, II, III)$ stand for the principal strains and $Q_{ji}$ are the components to a second-order orthogonal tensor, representing the coordinate transformation

$$[Q] = \begin{pmatrix} \cos(\theta_3)\cos(\theta_1) - \sin(\theta_3)\cos(\theta_2)\sin(\theta_1) & -\cos(\theta_3)\sin(\theta_1) - \sin(\theta_3)\cos(\theta_2)\cos(\theta_1) & \sin(\theta_3)\sin(\theta_2) \\ \sin(\theta_3)\cos(\theta_1) + \cos(\theta_3)\cos(\theta_2)\sin(\theta_1) & -\sin(\theta_3)\sin(\theta_1) + \cos(\theta_3)\cos(\theta_2)\cos(\theta_1) & -\cos(\theta_3)\sin(\theta_2) \\ \sin(\theta_2)\sin(\theta_1) & \sin(\theta_2)\cos(\theta_1) & \cos(\theta_2) \end{pmatrix}. \tag{6}$$

In the case of linear elasticity (2D or 3D materials) the Hooke's law can be applied and the stationary conditions of the strain energy density corresponding to objectives (2), (3) can be written as [11,12]

$$\frac{\partial U}{\partial \theta_i} = \frac{\partial (C_{ijkl}\varepsilon_{ij}\varepsilon_{kl})}{\partial \theta_i} = 0, \tag{7}$$

$$\frac{\partial (\hat{C}_{mnpq} Q_{im} Q_{kp} Q_{lq} \varepsilon_{ir} \varepsilon_{kl} - \Lambda_{rj} Q_{jn})}{\partial Q_{rn}} = 0, \tag{8}$$

where $\hat{C}_{mnpq}$ and $\Lambda_{rj}$ stand for the rotated constitutive matrix and Lagrange multipliers, respectively. The stationary conditions of the strain energy density corresponding to the current formulation (objective (4)) can be obtained by introducing the extended functional $J_*$ as

$$J_* = U + \lambda_m I_m \tag{9}$$

and equalizing its first variation to zero

$$\delta J_* = 0. \tag{10}$$

In (9) $\lambda_m$ and $I_m$ stand for the Lagrange multipliers and the constraints corresponding strain invariants, respectively. In details the constraint equations read

$$\begin{cases} I_1 = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} - \varepsilon_I - \varepsilon_{II} - \varepsilon_{III} = 0, \\ I_2 = \varepsilon_{11}\varepsilon_{22} + \varepsilon_{11}\varepsilon_{33} + \varepsilon_{22}\varepsilon_{33} - \varepsilon_{12}^2 - \varepsilon_{23}^2 - \varepsilon_{31}^2 - \varepsilon_I\varepsilon_{II} - \varepsilon_I\varepsilon_{III} - \varepsilon_{II}\varepsilon_{III} = 0, \\ I_3 = \varepsilon_{11}\varepsilon_{22}\varepsilon_{33} + 2\varepsilon_{12}\varepsilon_{23}\varepsilon_{31} - \varepsilon_{11}\varepsilon_{23}^2 - \varepsilon_{22}\varepsilon_{31}^2 - \varepsilon_{33}\varepsilon_{12}^2 - \varepsilon_I\varepsilon_{II}\varepsilon_{III} = 0. \end{cases} \tag{11}$$

It is assumed in (10) that the variations $\delta\varepsilon_{ij}$ are independent.

### 2.2. Necessary optimality conditions

The necessary optimality conditions can be derived from condition (10) as

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