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MITC technique extended to variable kinematic multilayered plate elements

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ABSTRACT

This paper considers the Mixed Interpolation of Tensorial Components (MITC) technique, which was originally proposed for Reissner–Mindlin type plates to develop shear locking free refined multilayered plate elements. Refined elements are obtained by referring to variable kinematic modelling in the framework of the Carrera Unified Formulation (CUF): linear, parabolic, cubic and fourth-order displacement fields in the thickness direction of the plate are used; both equivalent single layer (the multilayered plate is considered as an equivalent one-layer plate) and layer-wise (each layer is considered as an independent plate) variable descriptions are accounted for. Four-node elements are considered and a number of applications are developed for isotropic and multilayered anisotropic plates. Results related to the mixed interpolation of tensorial components are compared to the reduced and selective integration technique in the static and dynamic linear analysis. The numerical results show that the MITC technique maintains its effectiveness in the case of variable kinematic plate elements, hence the obtained elements are free from shear locking mechanisms. The capability of MITC to reduce/remove spurious modes is confirmed for refined multilayered elements.

1. Introduction

The shear locking phenomena in Reissner–Mindlin type finite elements (FEs) remains a milestone in finite elements analysis. Shear locking consists of a numerical difficulty (in the convergence rate sense) of reducing Reissner–Mindlin plate/shell elements to exact or Kirchhoff–Love solutions in the case of thin plate/shell analysis. The inclusion of both bending and shear stiffness in a unique rotational degree of freedom, in fact, makes it difficult to obtain zero-transverse-shear-energy in thin shell structures, as in physics. The mesh convergence rate becomes very small and a huge number of elements would be necessary to fulfill the zero-transverse-shear-energy constraint. A pioneering remedy, proposed in the late 1960's- early 1970's, is based on a reduced and/or selective numerical integration technique of transverse shear stiffness contributions. The reduced integration permits a significant increase of the convergence rate through the evaluation of the transverse shear strain energy in a plate integration point in which the corresponding shear strains are very small. The problem has been clearly explained and dealt with in detail in many FE books, such as those by Zienkiewic [1], Bathe [2] and Hughes [3], among others.

Unfortunately reduced integration techniques, since they introduce, 'by definition', an additional error into the evaluation of stiffness matrices, could lead to an increase in spurious and/or hourglass type 'non-physical' deformation modes which could destroy the related FE solutions. Such a drawback cannot be accepted especially in the case of nonlinear analysis, in which previous load-step solutions (which can be largely affected by spurious modes) are used to build the current equilibrium (e.g., with the Newton–Raphson method application). Non-convergent solution can be obtained in practical applications.

A remedy to this problem was introduced by many authors in the 80's. In particular, reference can be made to the work by Bathe and Dvorkin [4], even though similar works (earlier and/or later) by Mc Neal [5], Huang and Hinton [6] and Jang et al. [7] are known. In [4], the remedy was stated as Mixed Interpolation of Tensorial Components (MITC) and was first proposed for linear four node Q4 plate/shell elements. Subsequent work led to the extension to eight node Q8 elements. The MITC formulation permits the transverse shear locking phenomenon to be eliminated by introducing an independent FE approximation into the element domains for the transverse shear strains. Other authors have preferred to call this remedy as 'assumed shear strain field concept', as in [8]. A more rigorous mathematical justification of the MITC technique was established in the framework of the so-called Babuska–Brezzi

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conditions [9]. Further studies on the MITC method for Reissner–Mindlin problems were provided by Brezzi et al. [10] and Della Croce et al. [11,12].

Over the last decade, the first author has proposed the Carrera Unified Formulation (CUF) [13–16], for the analysis of layered structures. CUF permits a large variety of plate/shell theories as well as FEs, with variable kinematic and hierarchical properties, to be handled in a unified manner, on the basis of few so-called 'fundamental nuclei'. Refined Equivalent Single Layer (ESL) and Layer-Wise (LW) models have been used; the number of independent variables is maintained independent by the number of the constitutive layers N_l in the ESL models, while the unknown variables are independent in each layer in the LW models. The shear locking mechanism, in the related FE method, has been contrasted by using reduced/selective integration techniques, which lead to satisfactory results in the case of geometrically linear problems [17]. Extension to FE analysis of smart structures has been also provided in [18,19]. The present work aims at extending the MITC technique to variable kinematic models for the case of plate elements with four nodes. The aim is twofold: 1. to prove that the extension of MITC to higher-order (ESL and LW models) plate elements is feasible and 2. to show that such an extension is numerically efficient. The paper is organized as follows: Section 2 briefly recalls the original MITC technique for the case of Reissner–Mindlin plate theories; Section 3 shows the variable kinematic modelling in the CUF framework; Section 4 formulates the extension of the MITC technique to CUF by deriving the corresponding fundamental nuclei; Section 5 shows the obtained numerical results. The conclusions are drawn in the last section.

2. The MITC technique

A four node plate bending element, based on the Mindlin–Reissner plate theory and mixed interpolation, was proposed by Bathe and Dvorkin in [4]. The Mixed Interpolation Tensorial Component (MITC4) method calculates the transverse shear stresses σ_{xz} and σ_{yz} in a different manner from other tensorial components. Shear stresses are approximated in sample points of the domain Ω (see Fig. 1).

According to the Mindlin–Reissner theory, the displacement field is modelled as follows:

$$\begin{aligned} u_i &= u_i^0 + z\phi_i \quad i = x,y \\ u_z &= u_z^0 \end{aligned} \tag{1}$$

where $\phi_x,\ \phi_y$ are the rotations of the normals to the reference surfaces x–z and y–z, respectively; the apex^0 denotes a quantity calculated on Ω. By introducing the FE approximation, the displacement field is described by the relation:

$$\{u_x^0, u_y^0, u_z, \phi_x, \phi_y\} = \{N\}^T\{q\} \tag{2}$$

where $\{N\}$ denotes the vector of the interpolating Lagrange shape functions and $\{q\}$ is the vector of the nodal degrees of freedom. The dimension of $\{q\}$ is $5 \cdot nn$, where nn is the number of finite element nodes, in this case $nn = 4$.

Hooke's law can be conveniently written for a generic layer by splitting the bending (b) and shear (s) contributions:

$$\begin{Bmatrix} \{\sigma_b\} \\ \{\sigma_s\} \end{Bmatrix} = \begin{bmatrix} [Q_b] & [0] \\ [0] & [Q_s] \end{bmatrix} \begin{Bmatrix} \{\varepsilon_b\} \\ \{\varepsilon_s\} \end{Bmatrix} \tag{3}$$

The geometrical relations permit the strains ε_b and ε_s to be written as a function of the displacements, therefore:

$$\varepsilon_b = \begin{Bmatrix} u_{x,x}^0 \\ u_{y,y}^0 \\ u_{x,y}^0 + u_{y,x}^0 \end{Bmatrix} + z \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix}, \quad \varepsilon_s = \begin{Bmatrix} \phi_x + u_{z,x} \\ \phi_y + u_{z,y} \end{Bmatrix} \tag{4}$$

In matrix form:

$$\{\varepsilon_b\} = [B_b]q, \quad \{\varepsilon_s\} = [B_s]q \tag{5}$$

According to MITC4, the transverse shear strains are interpolated assuming the following shear strain field (see Fig. 1):

$$\{\varepsilon_s\} = \begin{Bmatrix} \{\varepsilon_{xz}\} \\ \{\varepsilon_{yz}\} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}(1+\xi)\varepsilon_{xz}^N + \frac{1}{2}(1-\xi)\varepsilon_{xz}^Q \\ \frac{1}{2}(1+\eta)\varepsilon_{yz}^P + \frac{1}{2}(1-\eta)\varepsilon_{yz}^M \end{Bmatrix} \tag{6}$$

where M, N, P and Q are sample points and $\xi,\ \eta$ are the natural coordinates in the plane of the element. The relations between the global coordinates $x,\ y$ and the natural coordinates $\xi,\ \eta$ are the following:

$$\begin{aligned} x &= \sum_{j=1}^{N_n} x_j N_j(\xi,\eta) \\ y &= \sum_{j=1}^{N_n} y_j N_j(\xi,\eta) \end{aligned} \tag{7}$$

with $-1 < \xi,\ \eta < 1$. N_j is the Lagrange shape function relative to node j.

In this way, the stiffness matrix K is written in two contributions (bending and shear):

$$\begin{aligned} [K] &= [K_b] + [K_s] \\ [K_b] &= \langle [B_b]^T [Q_b][B_b] \rangle; \quad [K_s] = \langle [B_s]^T [Q_s][B_s] \rangle \end{aligned} \tag{8}$$

where the following notation has been introduced:

$$\langle \ldots \rangle = \sum_{k=1}^{ns} \int_{V_k} (\ldots) dV_k \tag{9}$$

where ns is the number of layers and V_k is the volume of the layer k.

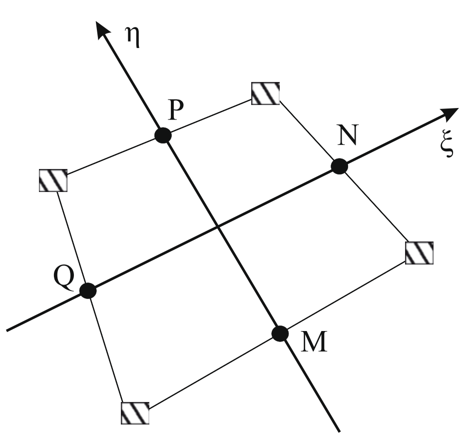

Fig. 1. Sample points (M,N,P,Q) to approximate the shear contribution in the MITC4 element.

3. Variable kinematic model via Carrera Unified Formulation

Carrera Unified Formulation (CUF) is a technique which handles a large variety of bi-dimensional models in a unified manner. According to CUF, the governing equations are written in terms of a few fundamental nuclei which do not formally depend on the order of expansion N used in the z direction and on the description of variables: ESL or LW. The application of a two-dimensional method for plates permits to express the unknown variables as a set of thickness functions depending only on the thickness coordinate z and the corresponding variables depending on the in plane coordinates x and y. So that, a generic variable, for instance the