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Peridynamic theory for progressive damage prediction in center-cracked composite laminates

B. Kilic, A. Agwai, E. Madenci *

Department of Aerospace and Mechanical Engineering, The University of Arizona, Tucson, AZ 85721, United States

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ABSTRACT

Numerical studies on the failure of composites have mostly employed the finite element method. However, it can be rather challenging to predict failure using this method. The assumption of lamina homogeneity is questionable when predicting failure, though it is suitable for stress analysis. Matrix cracking, fiber breakage, and delamination are inherent to the inhomogeneous nature of the composite, thus the assumption of material homogeneity taints the failure prediction. It is, therefore, essential that the inhomogeneous nature of the composites must be retained in the analysis in order to predict the correct failure modes. Hence, this study considers distinct properties of the fiber and matrix and their volume fractions and fiber orientations while modeling composite laminates. The peridynamic theory is employed to predict the damage in center-cracked laminates with different fiber orientations. The predictions from the peridynamic analysis agree with the experimental observations published in the literature.

1. Introduction

Unidirectional laminates are widely used in lightweight structures due to their excellent properties of high strength, high modulus, and low density. Moreover, they can be fabricated to produce parts with complex geometries and can be tailored to meet specific design requirements. However, damage initiation and its subsequent propagation are not understood as clearly as they are, for example, for metals because unidirectional composites consist of stiff fibers embedded into a matrix material, causing inhomogeneity. Furthermore, most composite structures have cutouts and notches arising from design and manufacturing requirements. Notches and cutouts reduce not only the strength of composites but also serve as potential failure sites for damage initiation. They also promote delamination or debonding, one of the most common failure modes in composites. Delamination may be caused by high stresses between laminae due to the mismatch of Poisson's ratios, impact, and structural discontinuities that generate local out-of-plane loads [1]. In addition to delamination, matrix cracking and fiber breakage can also be observed, which further complicates damage prediction.

In order to better understand failure mechanisms, Hallett and Wisnom [2] conducted experiments on double-edge-notched composite specimens made from E-glass. They reported the occurrence of matrix cracking before the ultimate failure for all specimens representing four different lay-ups when loaded in tension. Furthermore, it was reported that fiber failure initiated at the notch tip. Later, Green et al. [3] investigated the effect of scaling on the tensile strength of notched composites made from unidirectional carbon-fiber/epoxy pre-preg by considering the hole diameter and laminate thickness as independent variables. These experiments showed that failure mechanisms in composites can be very complex due to matrix cracking, fiber breakage, and delamination. Damage first initiated as isolated matrix cracks and delaminations at the hole edges, which then joined to form extensive delaminations. The overall trend indicated that the strength decreased with increase in specimen size. In another study, Sihn et al. [4] investigated the effect of thickness on laminated composites under uniaxial tension and fatigue loading for both unnotched and open-hole specimens by utilizing acoustic emission counts, X-ray photos, and c-scan images. It was observed that under static loading, the thicker specimens with open holes displayed a larger increase in strain near the hole edge than the thinner specimens with open holes. Furthermore, X-ray images revealed that damage also initiated from the hole edge, similar to observations by Green et al. [3]. In order to investigate the behavior of cracks, Wu [5] considered unidirectional fiberglass-reinforced Scotch-plies with center cracks oriented in the direction of the fibers. The plies had fibers in the 0° and 45° directions and were loaded in tension, pure shear, and combined tension and shear. In all three types of loading, it was observed that the crack propagated in a direction co-linear with the initial crack.

* Corresponding author. Tel.: +1 520 621 6113; fax: +1 520 621 8198.
E-mail address: madenci@email.arizona.edu (E. Madenci).

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Numerical studies on failure of notched composites have mostly utilized the finite element method to investigate the damage path and the initial failure load; such recent studies include works by Bogert et al. [6] and Satyanarayana et al. [7]. They predicted fiber and matrix damage in center-notched laminates for different lay-ups under tension. Both the experimental observations and numerical results suggest that damage initiation and crack propagation are dependent on ply orientation.

The use of the finite element method to predict failure can be quite challenging because remeshing may be required to make an accurate prediction and damage can only propagate in certain directions. Remeshing can be avoided by employing special elements, such as cohesive elements. However, these elements require a prior knowledge of the damage path, which might not be available. Unless these elements are correctly placed during model generation, the damage predictions may be erroneous. The difficulty in predicting failure using the finite element method comes from the mathematical form of the classical continuum mechanics equations. The equations of motion in classical continuum mechanics are in the form of partial differential equations that involve the spatial displacement derivatives; however, these derivatives are undefined when the displacements are discontinuous, such as across cracks or interfaces. Hence, failure prediction is posterior and requires special techniques.

Silling [8], realizing the aforementioned limitation, completely reformulated the basic equations of continuum mechanics. The resulting approach is known as the peridynamic theory. The main difference between the peridynamic theory and classical continuum mechanics is that the former is formulated using integral equations as opposed to derivatives of the displacement components. This feature allows damage initiation and propagation at multiple sites with arbitrary paths inside the material without resorting to special crack growth criteria. In the peridynamic theory, internal forces are expressed through nonlocal interactions between pairs of material points within a continuous body, and damage is a part of the constitutive model. Interfaces between dissimilar materials have their own properties and damage can propagate when and where it is energetically favorable for it to do so. The peridynamic theory was applied successfully by Colavito et al. [9,10] to predict damage in laminated composites subjected to low-velocity impact and static indentation. Askari et al. [11] and Xu et al. [12] also used peridynamic simulations to predict damage in laminates subjected to low-velocity impact and to notched laminated composites under biaxial loads.

The unidirectional composites usually consist of stiff fibers embedded into softer matrix material. Under the assumption of homogeneity, a lamina has orthotropic elastic properties. Even though this assumption is suitable for stress analysis, it becomes questionable when predicting failure. The major failure mechanisms in a composite are matrix cracking, fiber breakage, and delamination. These failure modes are inherent to the inhomogeneous nature of the composite, thus the homogeneous material assumption taints failure analysis. It is, therefore, evident that the inhomogeneous nature of the composites must be retained in the analysis to predict the correct failure modes. Each lamina with a different fiber orientation must be modeled as an inhomogeneous material with distinct matrix and fiber properties. Therefore, this study includes distinct properties of the fiber and matrix while employing the peridynamic theory for damage prediction in composite laminates with a center crack subjected to tension.

2. Peridynamic theory

The classical continuum theory is based on the assumption that a point in a continuum is influenced by the points that are located infinitesimally small distances away from it. This local nature of the classical continuum theory necessitates external failure criteria to predict crack growth since displacement derivatives are unbounded at the crack tip. Therefore, Eringen and Edelen [13], Kroner [14], and Kunin [15] introduced the nonlocal continuum theory. Because of the nonlocal nature of the theory, Eringen and Kim [16,17] showed that the stress field ahead of the crack tip is bounded as the crack tip is approached asymptotically, rather than unbounded as predicted by the classical continuum theory. Eringen and Kim [16] suggested a natural fracture criterion by equating maximum stress to the cohesive stress that holds the atomic bonds together. This criterion can be applied everywhere in continuous media without distinguishing discontinuities. Although their nonlocal continuum theory leads to finite stresses at the crack tips, the derivatives of the displacement field retained in the formulation have discontinuities arising from the presence of cracks. Another type of nonlocal theory, introduced by Kunin [15] and Rogula [18], circumvents this difficulty because it uses displacement fields rather than their derivatives. However, it is only given for a one-dimensional medium. More recently, Silling [8] independently reintroduced a nonlocal theory that does not require spatial derivatives – the peridynamic theory. Compared to the previous nonlocal theory by Kunin [15] and Rogula [18], the peridynamic theory is more general because it considers two- and three-dimensional media in addition to one-dimensional medium.

The peridynamic theory is concerned with the physics of a material body at a material point that interacts with all points within its finite distance, as shown in Fig. 1. As in the classical (local) continuum theory, the material points of a body are continuous, as opposed to the discrete nature of molecular dynamics. However, the main distinction between the peridynamic- and continuum-based methods is that the former is formulated using integral equations, as opposed to partial differential equations that include derivatives of the displacement components. Displacement derivatives do not appear in the peridynamic equations, which allows the peridynamic formulation to be valid everywhere whether or not displacement discontinuities are present. The peridynamic equation of motion at a reference configuration of position $\mathbf{x}$ and time t is given as

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \int_{\mathscr{R}} dV_{\mathbf{x}'} \mathbf{f}(\mathbf{u}(\mathbf{x},t), \mathbf{u}(\mathbf{x}',t), \mathbf{x}, \mathbf{x}', t) + \mathbf{b}(\mathbf{x},t), \tag{1}$$

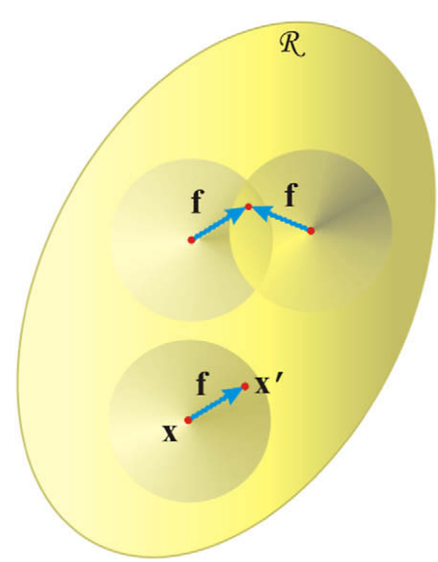

Fig. 1. Pairwise interaction of a material point with its neighboring points.