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Identification of effective elastic constants of composite plates based on a hybrid genetic algorithm

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ABSTRACT

Vibration testing combined with a numerical method is a potential alternative approach for determining elastic constants of materials. To obtain better results, the numerical method based on a hybrid genetic algorithm is proposed. As verified from available data in the literature, the repeatability and accuracy of the proposed inverse determination method are confirmed, and it also shows that the concept of effective elastic constants is workable. The proposed method is applied successfully to determine the effective elastic constants of a woven composite plate and two PCBs. Moreover, a systematical two-step procedure is proposed to make up the problem of missing natural frequencies or natural frequencies with large errors due to the limitation in the measurement. Hence, the applicability of the present method to identify the effective elastic constants of inhomogeneous composite plates is demonstrated.

1. Introduction

Composite materials, which consist of two or more different materials, have the characteristics of high modulus/weight and strength/weight ratios, excellent fatigue properties, and noncorroding behavior. These advantages encourage the extensive application of composite materials, for example, in sports and aerospace. The understanding of the mechanical behavior of composite materials is essential for their design and application. Composite materials are often heterogeneous, but they are presumed homogeneous from the viewpoint of macromechanics and only the averaged apparent mechanical properties are considered. For a transversely isotropic composite material, five elastic constants are necessary to describe the linearly elastic stress–strain relationship. If the geometry of the material could be considered as two-dimensional, four independent constants are necessary due to the assumption about the out-of-plane shear modulus or Poisson's ratio. These constants could be longitudinal Young's modulus, transverse Young's modulus, in-plane Poisson's ratio, and in-plane shear modulus.

The most common method to determine these constants is static testing. For composite materials, three types of specimens with different stacking sequences, which may be $[0^0_8]$, $[90^0_{16}]$, and $[\pm 45^0]_{2S,}$ are required in this method [1]. As well known, it is tedious and time consuming to prepare and test these three types of specimens. Also, the specimens prepared in laboratory may have different elastic constants from the structure in service fabricated by the same material because of different fabrication methods or curing processes. For some special composite plates such as sandwich panel or personal computer board (PCB), it is even more difficult to obtain their averaged apparent elastic constants by using static testing if these global properties are needed in the design and applications. Based on strain data, one possible approach for determining these constants is to combine a full-field [2] or local-field [3] measurement with a numerical method.

An alternative approach is to combine vibration testing with a numerical method. Natural frequencies and the corresponding mode shapes are generally obtained from vibration testing, and the elastic constants in the numerical model are updated such that the predicted dynamic properties fit the results from vibration testing. By this approach, the elastic constants can be determined in a single test, and they represent the global properties of a structure panel, not just local values. Also this approach is a nondestructive method, which may be suitable for quality control.

The determination of the four elastic constants has been focused in the analysis of thin plates. For example, Deobald and Gibson [4] analyzed a thin orthotropic plate with different boundary conditions and discovered that a plate with all boundaries free can obtain better results than that with one or more fixed edges. Ayorinde and Gibson [5] obtained the four elastic constants of a freely supported rectangular thin plate made from orthotropic materials with orthotropy ratio from one to thirteen. McIntyre and Woodhouse [6] identified both elastic and damping constants of thin orthotropic plates by measuring and analyzing the low modes of vibration. De Visscher et al. [7] also obtained the stiffness

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and damping properties of orthotropic composite plates by comparing the experimental modal parameters and the corresponding results from a numerical model in combination with a modal strain energy method. Recently, Lauwagie et al. [8] identified the in-plane elastic properties of the stainless substrate and thermal barrier coatings of beam-shaped layered specimens. To determine all the five independent elastic constants for transversely isotropic materials, thick plates are used for vibration testing because the effects of out-plane shear modulus (or out-plane Poisson's ratio) are more evident. Frederiksen [9] used single-layer plate theories, including the effects of transverse shear, to analyze the vibration of thick symmetrically laminated rectangular plates with all edges free. Ayorinde [10] incorporated through-the-thick shear and rotatory inertia in his method to obtain the three-dimensional elastic constants of a completely free orthotropic plate from experimental plate vibration data. Frederiksen [11] identified the elastic constants of thick orthotropic plates by the consideration of higher modes of natural frequencies linked with a numerical model based on a higher-order shear deformation theory. Daghia et al. [12] estimated the elastic constants of thick laminated plates by an estimator stemming from Bayes theorem and the minimum variance estimator.

In most the above references, the method used to search the suitable elastic constants is just a simple searching method, like the least-squares method. To obtain accurate and reliable elastic constants, it is necessary to use better searching methods. A subproblem approximation method was chosen to inversely determine the elastic constants by Hwang and Chang [13]. Ma and Lin [14] used the Simplex method for function minimization to obtain the effective elastic constants. An identification method combining experiment design, the response-surface function, and the finite analysis has been used for the elastic Constants of composite laminates [15–16]. The application of genetic algorithms (GA) for the identification of elastic constants has been demonstrated by Cunha et al. [17]. Lee and Kim [18] applied a multi-start global optimization algorithm together with a Rayleigh–Ritz technique to identify the elastic constants of elastically constrained laminated composite plates. Caillet et al. [19] presented two direct methods based on a modal representation and on an undulatory description of the vibrations that cover the low frequency domain and the high frequency domain.

Instead of a least-squares method or conventional calculus-based searching algorithms, a hybrid genetic algorithm [20], in which a simulated annealing (SA) mutation process and adaptive mechanisms are added to a real-parameter genetic algorithm, is used in the inverse determination method of this work to search for the elastic constants. Furthermore, this method will be applied to obtain the averaged apparent elastic constants, effective elastic constants hereafter, of some special composite plates, like woven composite laminates or PCBs. In these structures, the elastic constants of each material may have little application, and only the effective elastic constants are relevant. In addition, due to the limitation of vibration testing equipments, some natural frequencies may not be measured or may have large errors, which will create bias to the elastic constants identified. In this work, a two-step procedure is proposed and demonstrated to reduce the bias.

2. Experimental procedures

Impulse technique [21] is adopted for vibration testing, and modal analysis is used to extract the natural frequencies and mode shapes of the plates tested. The experimental set-up for impulse technique is shown in Fig. 1. The plate tested is suspended by suspension bands such that the direction of the plate vibration is normal to the suspension bands and the boundary conditions of all edges could be considered as free. The impulse excitation is accomplished by a PCB impulse hammer with a built-in force transducer. The response is detected by an accelerometer (PCB A353B68), whose mass is less than 2*g* such that its effect may be negligible. Both the input and output signals are converted to frequency domains by fast Fourier transformation in a two-channel signal analyzer, and then the frequency response functions are created. In total, 35 points are excited under this fixed response technique. After that, these frequency response functions are exported to a modal analysis program, SMS STARModal, in a personal computer. A polynomial function is used to fit the data of the frequency response function within a band under the assumption of single degree of freedom. After fitting all frequency response functions, one can obtain the natural frequencies, damping ratios, and mode shapes by the SMS STARModal.

3. Plate vibration model

For a transversely isotropic material, five independent elastic constants to describe its linear stress–strain relationship could be represented as

$$E_1, E_2 = E_3, \quad \nu_{12} = \nu_{13}, \nu_{23}, \quad G_{12} = G_{13}, \quad G_{23} = E_2/2(1+\nu_{23}) \tag{1}$$

where indicial notation is used. The longitudinal direction is denoted as 1 and the two transverse directions are denoted as 2 and 3. The Young's modulus is represented as E, the Poisson's ratio is represented as ν, and the shear modulus is represented as G. Among several methods to simplify a three-dimensional problem into a two-dimensional one, with four independent elastic constants, a popular one states

$$E_1, E_2 = E_3, \quad \nu_{12} = \nu_{13} = \nu_{23}, \quad G_{12} = G_{13}, \quad G_{23} = E_2/2(1+\nu_{23}) \tag{2}$$

When a plate is considered to be thin, these four independent elastic constants could be used to describe its linear stress–strain relationship.

For a symmetric laminate plate subjected to flexural motion, the moment-curvature relationships under classical lamination theory are as follows:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \tag{3}$$

where M is the moment resultant, k is the curvature, and D is the bending stiffness. If effective elastic constants are used to describe the vibration behavior of this symmetric laminate plate, these constants could be expressed by the following equations:

$$E_1 = \frac{12\det(D)}{(D_{22}D_{66} - D_{26}^2)h^3} \tag{4}$$

$$E_2 = \frac{12\det(D)}{(D_{11}D_{66} - D_{16}^2)h^3} \tag{5}$$

$$G_{12} = \frac{12\det(D)}{(D_{11}D_{22} - D_{12}^2)h^3} \tag{6}$$

$$\nu_{12} = \frac{D_{12}D_{66} - D_{16}D_{26}}{(D_{22}D_{66} - D_{26}^2)} \tag{7}$$

where h is the thickness of the plate and det(D) is the determinant of the bending stiffness matrix.

A commercially available finite element code (ANSYS 9.0) is utilized to model the undamped free vibration of a laminate plate. The natural frequencies and mode shapes of the plates are determined by the modal analysis in the ANSYS program. The whole domain is meshed by eight-node shell elements (SHELL93) with 6° of