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Fracture behaviour of brittle fibre-reinforced solids by a new FE formulation

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ABSTRACT

Fibre-reinforced materials are nowadays used in many different fields of applications due to their good load bearing capacity under general stress state and to their fracture and fatigue resistance. The use of reinforcing fibres, especially in brittle or quasi-brittle materials, generally allows a reduction of the manufacturing costs and to get more elevated performance with respect to traditional materials. In the present paper, a continuum finite element (FE) formulation to analyse fracture mechanics problems for fibre-reinforced and in plain brittle or quasi-brittle materials involving discontinuity of the displacement field is developed. By employing an energy approach, an appropriate stress field correction is introduced to simulate the displacement discontinuity (crack) and the mechanical effects of the fibres on the matrix material, and a new FE formulation is used to simulate the behaviour of such a class of materials. Finally, some simple 2D examples are presented in order to evaluate the capability of the proposed computational approach.

1. Introduction

The low values of fracture toughness for brittle materials (such as ceramics, concrete, and so on) can be increased by reinforcing them with fibres which increase tensile strength and connect the faces of the crack that can appear in the matrix material, with the beneficial effect to prevent its propagation. Several papers have been published on the mechanical characterisation of fibre-reinforced materials [1–6], while others studies have been conducted to investigate the behaviour of a tensile crack, situated in a fibre-reinforced composite material (see, for example, Refs. [7–16]).

Experimental observations show that the size of the crack bridging zone depends on the mechanical properties of the fibres and the matrix, and also on the adhesion strength of their connection. One of the most important issue is the knowledge of the deformation law of the fibres in the bridging zone and the possibility to account their possible pull-out from the matrix.

The numerical simulation of the mechanical behaviour of fibre-reinforced brittle solids presents several difficulties due to the need to describe crack formation and propagation in a heterogeneous material (matrix and fibres). As is well known, the formation of a discontinuity in a solid can produce computational instabilities which can show a non-uniqueness of the problem solution [17].

The simulation of the fracture process in brittle fibre-reinforced materials must consider at the same time the strain localisation that occurs in the cracked zone as well as the bridging effect produced by the fibres which cross the crack faces with the possibility of interface debonding; all the above mentioned phenomena leads to a highly non-linear problem which presents hard difficulties in computational simulation.

By using a new simple stress-based implementation of the mechanical effects of a discontinuous displacement field within an element [18] and a simple ad hoc mechanical model for fibre-reinforced materials with short and randomly dispersed fibres [19–21], a new FE formulation for such a class of materials is proposed in the present paper. In particular, this paper presents the stress-based discontinuous-like finite element and the mechanical model for fibre–matrix overall characterisation by taking into account at the same time partial or complete fibre debonding and elastic–plastic behaviour of the matrix material.

Finally, some simple 2D problems are presented to investigate the simulation capabilities of the proposed model by considering several values of the main mechanical parameters involved such as the fracture energy, the limit stress and the friction fibre–matrix interface shear stress.

2. Formulation of the problem

The description of the mechanical behaviour of fibre-reinforced brittle or quasi-brittle solids presents many difficulties since several mechanisms such as crack formation and propagation in a heterogeneous material – in which fibres can broke or slide with respect to the matrix – must be considered at the same time. The equilibrium of a body can be stated as:

$$\begin{aligned} &div\,\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \quad \text{in } B \\ &\mathbf{n} \cdot \sigma = \mathbf{t} \quad \text{on } \Gamma_t \\ &\mathbf{u} = \bar{\mathbf{u}} \quad \textbf{on } \Gamma_{\mathbf{u}} \end{aligned} \tag{1}$$

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Nomenclature

$A_m,\ A_f^p$ cross section area of the matrix and of the fibres belonging to the pth fibre phase in the REV, respectively
$\mathbf{b}$ body force vector field
B region occupied by a generic body
$\mathbf{B}$, $\mathbf{B}(\mathbf{x})$ generic compatibility matrix, compatibility matrix evaluated at the location $\mathbf{x}$ of the finite element, respectively
$\mathbf{B}^+(\mathbf{x}) = \sum_{i\in\Omega_e^+} B_i(\mathbf{x})$ sum of the compatibility matrices' values evaluated at location $\mathbf{x} \in \Omega_e^+$
$c,\ \phi$ cohesion and internal friction angle of the material
$\mathbf{C}(\mathbf{x}),\ \mathbf{C}_m(\mathbf{x}),\ \mathbf{C}'_m(\mathbf{x})$ generic elastic tensor, elastic tensor and tangent elastic tensor of the matrix, respectively, evaluated at location $\mathbf{x}$
$\mathbf{C}_{eq},\ \mathbf{C}'_{eq}$ homogenised elastic tensor and tangent homogenised elastic composite tensor, respectively
$a,\ D$ characteristic microscopic and macroscopic length, respectively
$\mathbf{D}$ element nodal discontinuity matrix
$\overline{E}_f$ Fibre's Young modulus
$E_m,\ E_t$ matrix Young's modulus and post-yielding stress–strain slope of the material
E'_f tangent Fibre's Young modulus
$f_t,\ f_c$ material's tensile and compressive strength, respectively
$\mathbf{f}_{e,u}^{(i)},\ \mathbf{f}_{e,ext}^{(i)}$ unbalanced and external force vector in the FE 'e' at step i, respectively
$F(\sigma_{ij},\ k_1,\ldots,k_n) = 0$ yield function
G_f fracture energy per unit crack surface
$H = E_t/(1 - E_t/E_m)$ yield hardening parameter
$H(\mathbf{x})$ heaviside jump function located at $\mathbf{x}$
G_m shear modulus of the matrix material
$\mathbf{k}$ unit vector parallel to the generic fibre axis
$\mathbf{i},\ \mathbf{j}$ unit vectors normal and perpendicular to the crack direction, respectively
$k_1,\ldots,k_n$ hardening parameters of the yield function
$2L_f^p$ length of the fibres belonging to the pth fibre phase
n_n number of nodes in a cracked FE lying in one of the two parts separated by the crack
$\mathbf{N}$, $\mathbf{N}(\mathbf{x})$ generic shape functions matrix of a finite element and shape functions matrix evaluated at the location $\mathbf{x}$, respectively
$\mathbf{N}^+(\mathbf{x}) = \sum_{i\in\Omega_e^+} N_i(\mathbf{x})$ sum of the shape functions evaluated at the location $\mathbf{x} \in \Omega_e^+$
p fibre's perimeter length
REV reference elementary volume
r_c crack surface roughness
$s_n,\ s_s$ correction factors to account for crack bridging and sliding effects, respectively
$s\left(\varepsilon_f^m\right),\ s\left(\overline{\varepsilon_f^m}\right)$ sliding function such that: $[\![\varepsilon_{f-m}]\!] = \varepsilon_f^m \cdot \left[1 - s\left(\varepsilon_f^m\right)\right]$ and mean value of the sliding function along a single fibre, respectively
S discontinuity locus in a cracked solid
$\mathbf{t}$ surface traction force vector field
$u_c,\ v_c$ normal and parallel component to the crack of the relative displacement vector across the crack, respectively
u_0 minimum opening crack displacement corresponding to the crack formation
$\mathbf{u},\ \overline{\mathbf{u}}$ displacement field and prescribed displacement field on Γ_u, respectively
V volume of the composite REV
$V_m,\ V_{f,p}$ volume of the matrix phase and of the pth fibre phase fraction present in the REV, respectively
$\mathbf{w}_c = \mathbf{u}_c + \mathbf{v}_c = \mathbf{u}_c\mathbf{i} + \mathbf{v}_c\mathbf{j}$ relative displacement vector across the crack faces
$\overline{W}$ mean fibre's elastic energy density
$\mathbf{x}$ generic position vector
$\boldsymbol{\delta}_d(\mathbf{x})$ discontinuous part of the displacement field
δ_s Dirac delta function located in S
$\boldsymbol{\delta}_{w,s} = \boldsymbol{\delta}_{u,s} + \boldsymbol{\delta}_{v,s}$ displacement vector discontinuity in a FE as a sum of the normal and tangential discontinuous components, respectively
$\boldsymbol{\delta}(\mathbf{x}),\ \overline{\boldsymbol{\delta}}(\mathbf{x}),\ \dot{\boldsymbol{\delta}}(\mathbf{x})$ displacement vector, continuous part and incremental displacement vector, respectively, in a generic solid or in a finite element
$[[\boldsymbol{\delta}(\mathbf{x})]] = \mathbf{w}(\mathbf{x})$ jump displacement vector in a generic point $\mathbf{x} \in S$ of a solid or in a finite element
$\boldsymbol{\varepsilon}(\mathbf{x})$ strain tensor evaluated at the location $\mathbf{x}$
$\varepsilon_f,\ \varepsilon_f^m,\ \overline{\varepsilon_f^m}$ uniaxial fibre's strain, uniaxial matrix strain and mean strain measured at the location and in the fibre's direction, respectively
$\overline{\varepsilon}_{p,f}$ limit strain for the pth fibre–matrix interface strain jump between the fibre and the matrix (parallel to fibre's axis) in the case of imperfect bond
$\boldsymbol{\varepsilon}^b(\mathbf{x}),\ \boldsymbol{\varepsilon}^u(\mathbf{x})$ bounded and unbounded part of the strain tensor evaluated at the location $\mathbf{x}$, respectively
$\varepsilon'^{el},\ \varepsilon'^{pl}$ elastic and plastic part of the strain rate tensor
$\dot{\lambda}$ plastic multiplier
$\boldsymbol{\sigma}(\mathbf{x})$ stress tensor evaluated at the location $\mathbf{x}$
$\sigma_c(u_c)$ stress–crack opening displacement relationship
$\boldsymbol{\sigma}$ stress tensor
$\sigma_f,\ \boldsymbol{\sigma}'_f$ Fibre's stress and stress rate tensor at the location of the generic fibre, respectively
σ_Y initial yield stress of the material
$\kappa(\mathbf{x}),\ \chi_p(\mathbf{x})$ point function denoting the presence of the matrix or of a fibre, belonging to the pth phase, at the location $\mathbf{x}$, respectively
$\mu = V_m/V$ REV matrix volume fraction
$\eta_{f,p} = V_{f,p}/V$ REV fibre volume fraction of the pth fibre phase, $p = 1,\ldots,\ q$
$\tau,\ \tau_{au},\ \tau_{fu}$ fibre–matrix interface shear stress, maximum and friction fibre–matrix interface shear stress, respectively
$\tau_c(u_c)$ shear stress–crack opening displacement relationship
$\Omega,\ \Omega^+,\ \Omega^-$ region occupied by a FE and regions separated by a crack in a FE, respectively
$\Gamma = \Gamma_t \bigcup \Gamma_u$ boundary of a generic solid
$\Gamma_t,\ \Gamma_u$ portions of the boundary on which tractions and displacements are prescribed, respectively

The microscopic incremental elastic constitutive relationship can be written as:

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) \cdot \boldsymbol{\varepsilon}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) \cdot (\nabla^s \otimes \mathbf{u}) \tag{2}$$

where $\boldsymbol{\varepsilon}$ is the strain tensor and the symbols "∇^s" and "$\otimes$" stand for symmetric gradient and tensor operator product, respectively. The fourth-order elastic tensor $\mathbf{C}(\mathbf{x})$ presents high variability with respect to the location where it is evaluated, as is shown by the dependence on the position vector $\mathbf{x}$. From a microscopic point of view, mechanical characteristics, such as the Young modulus $E(\mathbf{x})$ etc., in a heterogeneous material can be theoretically written in the following form:

$$E(\mathbf{x}) = E_m \cdot \kappa(\mathbf{x}) + E_f \cdot \chi_p(\mathbf{x}) \tag{3}$$

where the functions $\kappa(\mathbf{x}),\ \chi_p(\mathbf{x})$ can assume the following meaning: