Composite Structures 90 (2009) 418–427

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

An RVE-based micromechanical analysis of fiber-reinforced composites considering fiber size dependency

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ARTICLE INFO

Keywords:
Micromechanical elastic model
Fiber-reinforced composites
Fiber size dependency
Representative volume element (RVE)
Evolutionary interfacial debonding

ABSTRACT

An RVE-based micromechanical elastic damage model considering fiber size dependency is presented to predict the effective elastic moduli and interfacial damage evolution in fiber-reinforced composites. To assess the validity of the present model, the predictions based on the proposed micromechanical elastic model are compared with Hashin's theoretical bounds [Hashin Z. Analysis of properties of fiber composites with anisotropic constituents. J Appl Mech: Trans ASME 1979;46:543–50]. The proposed micromechanical elastic damage model is then exercised under uniaxial loading conditions to show the overall elastic damage behavior of the proposed micromechanical framework and to illustrate fiber size effect on the behavior of the composites. Moreover, comparisons between the present prediction and experimental data are made to further illustrate the capability of the proposed micromechanical framework for predicting the elastic damage behavior of fiber-reinforced composites.

1. Introduction

Various theoretical and numerical models for the prediction of behavior of fiber-reinforced composites have been proposed by many researchers and engineers. Since traditional continuum models based on the continuity, isotropy and homogeneity of materials cannot directly solve the problems of heterogeneous composites, micromechanics-based models have been extensively used to solve the problems on a finer scale and to relate the mechanics of materials to their microstructure [2,3]. Refer to Hill [4,5], Hashin [6], Willis [7], Mura [8], Zhao and Weng [9], Nemat-Nasser and Hori [10], Ju and Chen [11,12], Ju and Zhang [13], Lee [14], Lee and Simunovic [2], Lee and Liang [15], Lee and Kim [16], and Lee and Pyo [17] for micromechanics-based elastic composite modeling, and Ju and Chen [18], Ju and Tseng [19,20], Ju and Lee [21,22], Ju and Sun [23], Ju and Zhang [24], Lee and Simunovic [3], and Lee and Pyo [25] for micromechanics-based elastoplastic composite modeling.

The Eshelby's inclusion solution, which was derived based on the assumption that an inclusion is embedded in an unbounded infinite space, has been widely used in the existing micromechanics-based material models. However, the (classical) Eshelby's tensor is size independent with the prescribed uniform eigenstrains inside the inclusion and is valid only in case the size of the inclusion is relatively small in comparison with that of the representative volume element (RVE) [26]. Since the size of every RVE, in fact, is finite and the Eshelby's tensor needs to be dependent on the size of the inclusion [26,27].

Damage mechanisms in fiber-reinforced composites are complicated evolutionary phenomena and, therefore, the understanding of the damage is important for the investigation of the fiber-reinforced composites. Many numerical and experimental researches for the damage (or failure) phenomena have been conducted. Refer to van den Heuvel et al. [28–33], Zhou et al. [34], Teng [35], Aghdam et al. [36], Blassiau et al. [37,38] for details.

The present study aims to predict the effective elastic damage behavior of fiber-reinforced composites considering fiber size dependency. The size of the inclusion refers to the cross-section of fibers (not the fiber length), which is of certain related to their diameter. In the present study, a two-dimensional micromechanical formulation for the behavior of fiber-reinforced composites considering fiber size dependency is derived and a damage model for evolutionary interfacial debonding between fibers and the matrix is incorporated in the micromechanical formulation. The two-dimensional finite Eshelby's tensor [26,27] is adopted to consider the fiber size dependency and to solve problems in the finite domain. The Weibull's probabilistic function is incorporated into the micromechanical formulation to model evolutionary interfacial debonding between fibers and the matrix.

To assess the validity of the proposed micromechanical formulation, the predictions based on the proposed micromechanical elastic model are compared with Hashin's theoretical bounds [1]. The proposed micromechanical elastic damage model is then exercised under (transverse) uniaxial loading conditions to show the overall elastic damage behavior of the proposed micromechanical

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doi:10.1016/j.compstruct.2009.04.025

framework and to illustrate fiber size effect on the behavior of the composites. Furthermore, comparisons between the present prediction and experimental data reported by Wang et al. [39] and Li and Wisnom [40] are made to illustrate the capability of the proposed micromechanical framework for predicting the elastic damage behavior of fiber-reinforced composites.

2. Effective elastic damage behavior of circular fiber-reinforced composites considering the finite RVE

2.1. Recapitulation of the finite Eshelby's tensor

The summary of the finite Eshelby's tensor [26,27] for a circular inclusion embedded in a finite space is repeated here for completeness of the proposed micromechanics-based constitutive model. The finite Eshelby's tensor is adopted to model circular fibers in composites and is incorporated into the micromechanical framework. The finite Eshelby's tensor for a circular inclusion in a finite isotropic space subjected to the prescribed displacement or traction boundary conditions was systematically derived by Li et al. [26] and Wang et al. [27].

With consideration of a circular inclusion with radius r_0 embedded at the center of a finite, circular RVE with radius R as shown Fig. 1, the perturbed strain field ϵ_{ij}^{p} due to the existence of inclusion is related to the eigenstrain prescribed inside the inclusion as [8,41,42]

$$\epsilon_{ij}^{p}(\mathbf{x}) = S_{ijkl}^{F}(\mathbf{x})\epsilon_{kl}^{*} \tag{1}$$

where the eigenstrain ϵ_{kl}^{*} is given by

$$\epsilon_{kl}^{*}(\mathbf{x}) = \begin{cases} \epsilon_{kl}^{*}, & \text{if } \mathbf{x} \in \Omega_e \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The finite Eshelby's tensor for a circular inclusion in a circular finite RVE, denoted by S_{ijkl}^{F}, subjected to Dirichlet (displacement) boundary condition can be derived as [26]

$$\begin{aligned} S_{ijkl}^{F} = \frac{1}{8(1-\nu)} \Bigg\{ & \left[(4\nu-1)(1-\rho_0^2) + \frac{3\rho_0^2(\rho_0^2-1)(1-4\nu t^2)}{(4\nu-3)} \right] \delta_{ij}\delta_{kl}] \\ & + \left[(3-4\nu)(1-\rho_0^2) + \frac{3\rho_0^2(\rho_0^2-1)(2t^2-1)}{(4\nu-3)} \right] (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \\ & + \left[\frac{12(2\nu-1)\rho_0^2(\rho_0^2-1)}{(4\nu-3)} \right] t^2 \delta_{ij} r_k r_l \Bigg\} \end{aligned} \tag{3}$$

where δ_{ij} signifies the Kronecker delta, ν is the Poisson's ratio of the matrix, $\rho_0 = r_0/R, \rho = r_0/|\mathbf{x}|$, $t = \rho_0/\rho$, and $r_i(\mathbf{x}) = \frac{x_i}{|\mathbf{x}|}$ $(i = 1, 2)$ are the components of the unit vector in the direction of the position vector $\mathbf{x}$ [26,27].

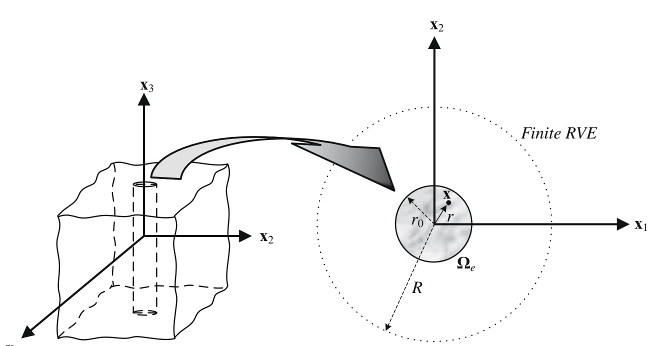

Fig. 1. A circular inclusion embedded in a finite representative volume element (RVE) (see also Li et al. [26]).

In case of the Neumann (traction) boundary condition, the finite Eshelby's tensor can be derived as [27]

$$\begin{aligned} S_{ijkl}^{F} = \frac{1}{8(1-\nu)} \{ & [(4\nu-1)(1-\rho_0^2) - 3\rho_0^2(1-\rho_0^2)(1-4\nu t^2)] \delta_{ij}\delta_{kl} \\ & + [(3-4\nu) + \rho_0^2 + 3\rho_0^2(1-\rho_0^2)(1-2t^2)] (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})] \\ & + [12(1-2\nu)\rho_0^2(1-\rho_0^2)t^2] \delta_{ij} r_k r_l \} \end{aligned} \tag{4}$$

To simplify Eqs. (3) and (4), the averaging process over an inclusion Ω_e is applied to the equations (cf. [26]).

$$\begin{aligned} \langle t^2 \rangle_{\Omega_e} &= \frac{1}{A_{\Omega_e}} \int_0^{2\pi} \int_0^{r_0} r t^2 dr d\theta = \frac{\alpha}{2}, \\ \langle t^2 r_k r_l \rangle_{\Omega_e} &= \frac{1}{A_{\Omega_e}} \int_0^{2\pi} \int_0^{r_0} r t^2 r_k r_l dr d\theta = \frac{\alpha}{4} \delta_{kl} \end{aligned} \tag{5}$$

where $\langle \cdot \rangle_{\Omega_e}$ signifies the average over an inclusion Ω_e, $A_{\Omega_e} (= \pi r_0^2)$ denotes the area of the inclusion, and $\alpha = \rho_0^2 (< 1)$. Finally, the finite Eshelby's tensor in Eqs. (3) and (4) is simplified by applying the aforementioned average process given in Eq. (5) as [26,27]

$$S_{ijkl}^{F} = S_1 \delta_{ij}\delta_{kl} + S_2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \quad i,j,k,l = 1,2 \tag{6}$$

where the parameters S_1 and S_2 for the Dirichlet boundary condition are given by

$$\begin{aligned} S_1 &= \frac{1-\alpha}{8(1-\nu)} \left[(4\nu-1) + \frac{3\alpha(1-\alpha)}{3-4\nu} \right], \\ S_2 &= \frac{1-\alpha}{8(1-\nu)} \left[(3-4\nu) - \frac{3\alpha(1-\alpha)}{3-4\nu} \right] \end{aligned} \tag{7}$$

and the parameters S_1 and S_2 for the Neumann boundary condition are given by

$$\begin{aligned} S_1 &= \frac{1-\alpha}{8(1-\nu)} [(4\nu-1) + 3\alpha(\alpha-1)], \\ S_2 &= \frac{1}{8(1-\nu)} [(3-4\nu) + \alpha(3\alpha^2 - 6\alpha + 4)] \end{aligned} \tag{8}$$

in which $\alpha = \rho_0^2$. It is noted from Eqs. (7) and (8) that the finite Eshelby's tensor depends on the Poisson's ratio of the matrix and the ratio of the size of the inclusion to that of the RVE. As the ratio of the size of the inclusion to that of the RVE becomes smaller ($\alpha \to 0$), the finite Eshelby's tensor in Eq. (6) naturally converges to the following infinite Eshelby's tensor

$$S_{ijkl}^{\infty} = \frac{4\nu-1}{8(1-\nu)} \delta_{ij}\delta_{kl} + \frac{3-4\nu}{8(1-\nu)} (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \quad i,j,k,l = 1,2 \tag{9}$$

where S_{ijkl}^{∞} is the Eshelby's tensor for a circular inclusion embedded in an isotropic linear elastic and infinite matrix. Refer to Mura [8], Ju and Zhang [13], Li et al. [26], Wang et al. [27] for more details of S_{ijkl}^{∞} and S_{ijkl}^{F}.

2.2. Evolutionary damage model

Let us start by considering a two-phase composite consisting of an elastic matrix (phase 0) with bulk modulus κ_0 and shear modulus μ_0 and unidirectionally aligned, perfectly bonded elastic fibers (phase 1) with bulk modulus κ_1 and shear modulus μ_1 in the matrix. The evolutionary damage between fibers and the matrix may occur as transverse loadings or deformations continue to increase. After the evolutionary damage between fibers and the matrix, the debonded circular fibers may lose the load-carrying capacity and are assumed to completely debonded fibers (circular voids) (phase 2) for simplicity within the present framework. It is assumed that circular fibers embedded in the matrix are long in the fiber direction (the X_3-axis) and the fiber and void orientation in the 1–2 plane is random. With the help of the finite Eshelby's tensor proposed by