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Three-dimensional free vibration analysis of thick laminated annular sector plates using a hybrid method

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ABSTRACT

An accurate and efficient solution procedure based on the three-dimensional elasticity theory for the free vibration analysis of thick laminated annular sector plates is presented. Plates with simply supported radial edges and arbitrary boundary conditions on their circular edges are considered. In order to accurately model the variation of material properties across the thickness, the layerwise theory is used to approximate the displacement components in this direction. Then, employing the Hamilton's principle together with the modal analysis, through-the-thickness and circumferential discretized form of the equations of motion and the related boundary conditions are obtained. Finally, the differential quadrature method (DQM) as an efficient and accurate numerical method is applied to discretize the resulting variable coefficients differential equations in the radial direction. The fast rate of convergence of the method is demonstrated and to show its high accuracy, comparison studies with the available results in the literature are made. Finally, some new results are prepared, which can be used as benchmark solutions for future works.

1. Introduction

Laminated annular sector plates have found wide applications as structural members in aerospace, marine and other industries. In comparison with research works on the free vibration analyses of isotropic sector plates, some recent works of which are cited in Refs. [1–10], only limited references can be found for laminated composite annular sector plates [11–13].

Srinivasan and Thiruvenkatachari [11] presented a free vibration analysis of laminated annular sector plates with clamped edges, which include the in-plane and rotary inertia. The frequencies are obtained by using an integral equation technique. Sharma et al. [12] studied the vibration and stability of moderately thick laminated sector plates made up of cylindrically orthotropic layers using the Chebyshev polynomials. Their formulation was based on the first-order shear deformation theory (FSDT). More recently, using a sector *p*-element, Houmat [13] studied the nonlinear free vibration of laminated sector plates based on the FSDT.

Most of the previous studies on the free vibration of isotropic and composite annular sector plates are based on the two-dimensional (2D) classical or the shear deformable plate theories [1–7,11–13]. However, from the comparison study with the 2D solutions, it was found that the 2D theories can only predict the flexural modes and they are unable to yield the thickness twist and shear modes [8]. The ability to detect these modes is an important attribute of the three-dimensional elasticity method. In addition, to more accurately determine the out-of-plane natural modes, the effects of the transverse shear deformation and rotary inertia should be accurately included in the formulations. Hence, the development of the three-dimensional based approaches becomes essential.

On the other hand, only limited works can be found in the published literature about the vibrations of annular sector plates based on the three-dimensional (3D) elasticity theory [7–10], which are limited to isotropic annular sector plates. Mizusawa [7] used the finite prism method to study the free vibration characteristic of annular sector plates with radial edges simply supported based on the 3D elasticity theory. Liew et al. [8] used the orthogonal polynomials as the admissible functions of the Ritz method to analyze the 3D free vibration of thick annular sector plates. Houmat [9] employed the 3D hierarchical finite element method to investigate the 3D free vibration of annular sector plates. More recently, Zhou et al. [10] analyzed the 3D free vibration of thick annular sector plates using the Chebyshev polynomials as the admissible functions of the Ritz method.

To the best of author's knowledge, the three-dimensional free vibration analysis of laminated annular sector plates is not investigated yet. Due to complexity of the governing equations, if it is not

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impossible to find an exact solution for the free vibration of thick laminated annular sector plates based on the three-dimensional elasticity theory, it is very difficult to find such a solution. In the previous 3D studies of isotropic sector plates, the approximate methods such as the finite prism method [7], the finite element method [9] and the Ritz methods [8,10] were used. In this work, using the layerwise theory and the differential quadrature method (DQM) in conjunction with the modal analysis, a computationally efficient and accurate approach for the titled problem is developed.

The layerwise theory is a refined theory that can take into account the thickness effects with minimum computational cost [14–16]. Unlike the equivalent single layer theories, the layerwise theories assume separate displacement field expansions within each subdivision. Hence, the layerwise theory provides a kinematically correct representation of the strain field in discrete layers [14–16]. This is an advantage of the layerwise theories in comparison with the higher-order shear deformation theories. Since the shear strains are discontinuous, this leaves the possibility of the continuous transverse stresses between adjacent layers in the layerwise theory. Differential quadrature method (DQM) as an alternative numerical technique was used to solve the different structural problems. The development and its applications can be found in a review paper by Bert and Malik [17] and also in the recent research works [18–22].

Here, based on the three-dimensional elasticity theory and by employing the Hamilton's principle together with the modal analysis, through-the-thickness and circumferential discretized form of the equations of motion and the related boundary conditions are obtained. Then, the differential quadrature method (DQM) as an efficient and accurate numerical method is applied to discretize the resulting variable coefficients differential equations in the radial direction. After demonstrating the fast rate of convergence of the method and its high accuracy, some new results for the free vibration of laminated annular sector plates are prepared, which can be used as benchmark solutions for future works.

2. Layerwise theory

Consider a laminated thick annular sector plates composed of N_L perfectly bonded orthotropic layers of width b, total thickness h, sector angle θ_0, inner radius R_i and outer radius R_o (Fig. 1). Each lamina is considered to be cylindrically orthotropic with the fiber orientation being either in the radial or the circumferential directions. The fiber angle is measured from the radial axis.

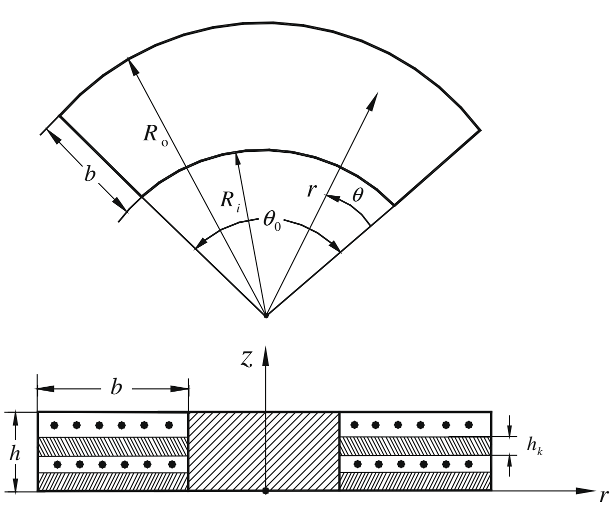

Fig. 1. An arbitrary laminated thick annular sector plate.

Based on the three-dimensional theory of elasticity, the linearized strain–displacement relations are,

$$\varepsilon_{rr}=\frac{\partial u}{\partial r},\quad \varepsilon_{\theta\theta}=\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{u}{r},\quad \varepsilon_{zz}=\frac{\partial w}{\partial z},\quad \gamma_{r\theta}=\frac{\partial v}{\partial r}+\frac{1}{r}\left(\frac{\partial u}{\partial \theta}-v\right),$$
$$\gamma_{rz}=\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z},\quad \gamma_{\theta z}=\frac{1}{r}\frac{\partial w}{\partial \theta}+\frac{\partial v}{\partial z} \tag{1}$$

where ε_{ii} and γ_{ij} ($i, j = r, \theta, z$ *with* $i \neq j$) are the normal and the shear components of the strain tensor; u, v and w are the displacement components along the radial (r), the circumferential (θ) and the thickness (z) directions, respectively.

In the equivalent single-layer theory of laminated plates, which is the basis of the two-dimensional theories, the assumption of the plane stress in the thickness direction of each lamina is used. But, here, this assumption is removed and to derive the stress strain relations at an arbitrary point of a lamina, the three-dimensional constitutive relations is used, which is,

$$\begin{Bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\\ \sigma_{zz}\\ \sigma_{z\theta}\\ \sigma_{rz}\\ \sigma_{r\theta}\end{Bmatrix}=\begin{bmatrix}\bar{C}_{11}&\bar{C}_{12}&\bar{C}_{13}&0&0&\bar{C}_{16}\\ \bar{C}_{12}&\bar{C}_{22}&\bar{C}_{23}&0&0&\bar{C}_{26}\\ \bar{C}_{13}&\bar{C}_{23}&\bar{C}_{33}&0&0&\bar{C}_{36}\\ 0&0&0&\bar{C}_{44}&\bar{C}_{45}&0\\ 0&0&0&\bar{C}_{45}&\bar{C}_{55}&0\\ \bar{C}_{16}&\bar{C}_{26}&\bar{C}_{36}&0&0&\bar{C}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ \gamma_{z\theta}\\ \gamma_{rz}\\ \gamma_{r\theta}\end{Bmatrix} \tag{2}$$

where σ_{ij} ($i, j =$ r, θ, z) are the stress tensor components; $[\bar{C}]=[T][C][T]^T$ in which $[C]$ is the principle material stiffness matrix of the lamina and $[T]$ represents the transformation matrix [23].

To develop a layerwise model for the laminated plates that poses full three-dimensional modeling capability, a displacement field that accounts for all the strain components in a kinematically correct manner must be assumed. Specifically, the in-plane strain ε_{ii} ($i = r, \theta$) should be continuous while the transverse strain γ_{iz} ($i = r, \theta$) should be piecewise continuous through the laminate thickness. For this purpose, the plate is divided into N_m ($\geqslant N_L$) mathematical layers. Then, the displacement components in each subdivision are expressed in terms of the nodal values in the thickness direction. Using the concept of the global interpolation functions, one can express the displacement components at an arbitrary material point of the plate as

$$U(r,\theta,z,t)=\sum_{i=1}^{N_z}U_i(r,\theta,t)\varphi_i(z),\quad v(r,\theta,z,t)=\sum_{i=1}^{N_z}V_i(r,\theta,t)\varphi_i(z),$$
$$w(r,\theta,z,t)=\sum_{i=1}^{N_z}W_i(r,\theta,t)\varphi_i(z) \tag{3}$$

where φ_i denotes the global interpolation function of the node 'i' in the z-direction; U_i, V_i and W_i represent the displacement components of node 'i' (defined by $z = z_i$) in the r-, θ- and z-directions, respectively. Also, N_z stands for the total number of nodes through the thickness of the plate, which depends on the number of mathematical layers (N_m) and nodes per layer in the thickness direction (N_{PL}) as $N_z = N_m(N_{PL} - 1) + 1$.

In the present study, one-dimensional Lagrange interpolation functions are used in each mathematical layer and hence the global interpolation function $\varphi_i(z)$ can easily be obtained. Any desired degrees of displacement variation through the thickness are easily obtained by either adding more subdivisions (mathematical layers) or using the higher-order Lagrangian interpolation polynomials through the thickness.

Substituting the displacement components from Eq. (3) into Eq. (1), the results read,