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Free and forced vibration of a functionally graded beam subjected to a concentrated moving harmonic load

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ABSTRACT

In this paper, free vibration characteristics and the dynamic behavior of a functionally graded simply-supported beam under a concentrated moving harmonic load are investigated. The system of equations of motion is derived by using Lagrange's equations under the assumptions of the Euler–Bernoulli beam theory. Trial functions denoting the transverse and the axial deflections of the beam are expressed in polynomial forms. The constraint conditions of supports are taken into account by using Lagrange multipliers. It is assumed that material properties of the beam vary continuously in the thickness direction according to the exponential law and the power-law form. In this study, the effects of the different material distribution, velocity of the moving harmonic load, the excitation frequency on the dynamic responses of the beam are discussed. Numerical results show that the above-mentioned effects play very important role on the dynamic deflections of the beam.

1. Introduction

Functionally graded materials (FGMs) were proposed in Japan in around 1984–1985 during a space plane project. FGMs consisting of metallic and ceramic components are well-known to improve the properties of technical devices such as thermal barrier systems, because cracking or delamination, which are often observed in conventional two-layer composite systems, are avoided due to the smooth transition between the properties of the components in FGMs. Also, the gradual transition between the heat and corrosion resistant outer layer (often made of a ceramic material) and the tough metallic base material increases in most cases the lifetime of the component. In nature, we can see examples of FGMs in bamboo, bones, and teeth. In each case, there is a hard wear-resistant exterior that smoothly transitions to a soft interior. FGMs are of interest for a wide range of applications: thermal barrier coatings for turbine blades (electricity production), armor protection for military applications, fusion energy devices, biomedical materials including bone and dental implants, space/aerospace industries, automotive applications, etc.

Studies devoted to understand the static and dynamic behavior of the functionally graded beams, plates and shells have gained importance in the last decades because of the wide application areas of FGMs. Although homogeneous or classical composite beams subjected to moving loads have been widely studied (see Refs. [1–20]), the research effort devoted to vibration of FG beams under the moving loads has been very limited. Sankar [21] gave an elasticity solution based on the Euler–Bernoulli beam theory for functionally graded beam subjected to static transverse loads by assuming that Young's modulus of the beam vary exponentially through the thickness. Chakraborty et al. [22] proposed a new beam finite element based on the first-order shear deformation theory to study the thermoelastic behavior of functionally graded beam structures. In [22], static, free and wave propagation analysis are carried out to examine the behavioral difference of functionally graded material beam with pure metal or pure ceramic. Chakraborty and Gopalakrishnan [23] analyzed the wave propagation behavior of FG beam under high frequency impulse loading, which can be thermal or mechanical, by using the spectral finite element method. Aydogdu and Taskin [24] investigated the free vibration behavior of a simply supported FG beam by using Euler–Bernoulli beam theory, parabolic shear deformation theory and exponential shear deformation theory. Zhong and Yu [25] presented an analytical solution of a cantilever FG beam with arbitrary graded variations of material property distribution based on two-dimensional elasticity theory. Ying et al. [26] obtained the exact solutions for bending and free vibration of FG beams resting on a Winkler–Pasternak elastic foundation based on the two-dimensional elasticity theory by assuming that the beam is orthotropic at any point and the material properties vary exponentially along the thickness direction. Kapuria et al. [27] presented a finite element model for static and free vibration responses of layered FG beams using an efficient third order zigzag theory for estimating the effective modulus of elasticity, and its experimental validation for two different FGM systems under various boundary conditions. Yang and Chen

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Nomenclature

A	area of the cross section
A_{xx}	extensional rigidity
A_n	time-dependent generalized coordinate of the transverse displacements
b	width of the cross section
B_n	time-dependent generalized coordinate of the axial displacements
B_{xx}	coupling rigidity
$c(t)$	unit step function
D_{xx}	bending rigidity
E_L	Young's modulus of the lower surface of the beam
E_U	Young's modulus of the upper surface of the beam
$F(t)$	generalized load vector
h	depth of the cross section
I	moment of inertia of the cross section
I_A, I_B, I_D	inertial coefficients
J	functional of the problem
J^*	Lagrangian functional of the problem
k	power-law exponent
K	stiffness matrix
$K_5^S \ldots K_8^S$	matrices due to Lagrange multipliers
K_e	kinetic energy of the beam
L	length of the beam
M	mass matrix of the beam
N	number of the terms in the displacement functions
P_L	material properties of the lower surface of the beam
P_U	material properties of the upper surface of the beam
$q(t)$	generalized coordinates
$Q(t)$	concentrated moving harmonic load
Q_0	amplitude of the moving harmonic load
t	time
t_1	time the load $Q(t)$ comes onto the beam
t_2	time the load $Q(t)$ leaves the beam
u	axial displacement
u_0	axial displacement of any point on the neutral axis
U	strain energy of the beam
v	velocity of the moving harmonic load
V	potential of the external loads
w	transverse displacement
w_0	transverse displacement of any point on the neutral axis
x	x coordinate
x_Q	location of the concentrated moving harmonic load
z	z coordinate
$\beta_1, \beta_2, \beta_3$	Lagrange multipliers
ε_{xx}	normal strain
λ	dimensionless natural frequency
ρ_L	mass density of the lower surface of the beam
ρ_U	mass density of the upper surface of the beam
σ_{xx}	normal stress
Ω	excitation frequency of the moving harmonic load
ω	natural frequency
ζ	a dimensionless parameter

[28] studied the free vibration and elastic buckling of FG beams with open edge cracks by using Euler–Bernoulli beam theory. Li [29] proposed a new unified approach to investigate the static and the free vibration behavior of Euler–Bernoulli and Timoshenko beams. In a recent study by Yang et al. [30], free and forced vibrations of cracked FG beams subjected to an axial force and a moving load were investigated by using the modal expansion technique.

To the best of the authors' knowledge, this is the first attempt on the vibration of a FG beam subjected to a concentrated moving harmonic load. In the study [30], it is assumed that material properties of the beam vary exponentially in the thickness direction and the moving load is a constant moving load.

The aim of this paper is to investigate the free and forced vibration of a functionally graded simply-supported beam subjected to a concentrated moving harmonic load. In the dynamic responses of the beam, the space-dependent functions are chosen as the polynomial functions. The system of equations of motion is derived by using Lagrange's equations under the assumptions of the Euler–Bernoulli beam theory. The constraint conditions of supports are taken into account by using Lagrange multipliers. It is assumed that material properties of the beam vary continuously in the thickness direction according to the exponential law and the power-law form. Equations of motion are solved by using the implicit time integration Newmark-β method, and then displacements, velocities and accelerations of the beam at the considered point and time are determined. In this study, the effects of the different material distribution, velocity of the moving harmonic load, the excitation frequency on the dynamic responses of the beam are discussed. Numerical results show that the above-mentioned effects play very important role on the dynamic deflections of the beam.

2. Theory and formulations

A functionally graded simply-supported beam of length L, width b, thickness h, with co-ordinate system (O x y z) having the origin O is shown in Fig. 1. The beam is subjected to a concentrated moving harmonic load, $Q(t)$, which moves in the axial direction of the beam with constant velocity.

In this study, it is assumed that material properties of the beam, i.e., Young's modulus E and mass density ρ, vary continuously in the thickness direction (z-axis) according to the exponential law and the power-law form. Therefore, the material properties are functions of the z coordinate, namely $E = E(z)$ and $\rho = \rho(z)$. The exponential law is given by [31],

$$P(z) = P_U \exp[-\delta(1 - 2z/h)], \quad \delta = \frac{1}{2} \log\left(\frac{P_U}{P_L}\right) \tag{1}$$

and the power-law introduced by [32] can be written as

$$P(z) = (P_U - P_L)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_L \tag{2}$$

where P_U and P_L are the corresponding material properties of the upper and the lower surfaces of the beam, and k is the non-negative power-law exponent which dictates the material variation profile through the thickness of the beam. It is evident from Eqs. (1) and (2) that when $z = -h/2$, $P = P_L$ and when $z = h/2$, $P = P_U$.

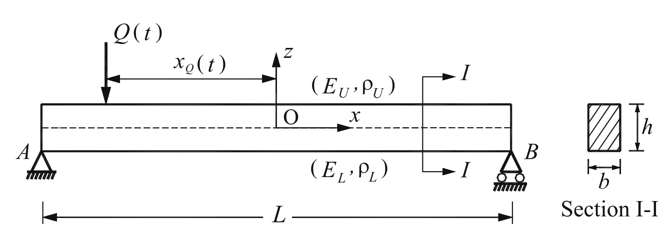

Fig. 1. A functionally graded simply-supported beam subjected to a concentrated moving harmonic load.