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Three-dimensional free vibration analysis of thick functionally graded plates on elastic foundations

P. Malekzadeh *

Department of Mechanical Engineering, Persian Gulf University, Bushehr 75168, Iran
Center of Excellence for Computational Mechanics, Shiraz University, Shiraz, Iran

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ABSTRACT

The research works on the three-dimensional (3D) free vibration analyses of functionally graded (FG) plates are limited to plates with simply supported boundary conditions and without elastic foundations. Hence, the free vibration analysis of thick FG plates supported on two-parameter elastic foundation is presented. The formulations are based on the three-dimensional elasticity theory. Plates with two opposite edges simply supported and arbitrary boundary conditions at other edges are considered. A semi-analytical approach composed of differential quadrature method (DQM) and series solution is adopted to solve the equations of motions. The material properties change continuously through the thickness of the plate, which can vary according to power law, exponentially or any other formulations in this direction. The fast rate of convergence of the method is demonstrated and comparison studies are carried out to establish its very high accuracy and versatility. Some new results for the natural frequencies of the plate are prepared, which include the effects of elastic coefficients of foundation, boundary conditions, material and geometrical parameters. The new results can be used as benchmark solutions for future researches.

1. Introduction

Functionally graded materials (FGMs) are special composite whose thermo-mechanical properties have smoothly and continuously spatial variation due to a continuous change in composition, in morphology, in microstructure, or in crystal structure. FGMs posses various advantages over the conventional composite laminates, such as smaller thermal stresses, stress concentrations, attenuation of stress waves, etc. Therefore, FGMs have received wide applications as structural components in modern industries such as mechanical, aerospace, nuclear, reactors, and civil engineering.

The vibration characteristic of thick plates made of FGMs is of great interest for engineering design and manufacture. Most of the previous studies on the free vibration of FG plates are based on the two-dimensional theories, such as the classical plate theory, the first and the higher order shear deformation plate theories [1–9]. These plate theories neglect transverse normal deformations, and generally assume that a plane stress state of deformation prevails in the plate. These assumptions may be appropriate for thin plates, but may not give good results for thick plates with length/thickness equal to 5 or less [10]. Yang and Shen [1] used the classical plate theory to study the free and forced vibration of functionally graded rectangular thin plates subjected to initial in-plane stresses and rested on elastic foundations. Cheng and Kitipornchai [2] proposed a membrane analogy to derive an exact explicit eigenvalues for vibration and buckling of simply supported FG plates resting on elastic foundation using the first order shear deformation theory (FSDT). Batra and Jin [3] used the FSDT coupled with the finite element method (FEM) to study free vibrations of a functionally graded (FG) anisotropic rectangular plate. Cheng and Batra [4] used Reddy's third-order plate theory to study steady state vibrations and buckling of a simply supported functionally gradient isotropic polygonal plate resting on a Pasternak elastic foundation and subjected to uniform in-plane hydrostatic loads. Yang and Shen [5] investigated the free and forced vibration analyses of initially stressed functionally graded plates in thermal environment. Theoretical formulations are based on Reddy's higher order shear deformation plate theory and include the thermal effects due to uniform temperature variation. Qian et al. [6] analyzed static deformations, free and forced vibrations of a thick rectangular functionally graded elastic plate by using a higher order shear and normal deformable plate theory (HOSNDPT) and a meshless local Petrov–Galerkin (MLPG) method. Ferreira et al. [7] used the global collocation method and approximate the trial solution with multiquadric radial basis functions to analyze the free vibration of functionally graded plates based on the first and the third-order

* Tel.: +98 771 4222150; fax: +98 771 4540376.
E-mail addresses: p_malekz@yahoo.com, malekzadeh@pgu.ac.ir

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shear deformation plate theories. Roque et al. [8] performed the free vibration analysis of functionally graded plates by using the multiquadric radial basis function method and a higher order shear deformation theory. More recently, Matsunaga [9] analyzed the natural frequencies and buckling stresses of FG plates using a higher order shear deformation theory which are based on the through the thickness series expansion of the displacement components.

On the other hands, the research works on the three-dimensional (3D) free vibration analyses of FG plates are limited to plates with simply supported boundary conditions and without elastic foundations [10–13]. In these works, in order to satisfy the edge conditions, a trigonometric series solution is assumed in the plane of the plate to transform the partial differential equations into a set of ordinary differential equation in the thickness direction. Then, different procedures such as power series solution [10], state space method [11,12], and FEM [13] are employed to discretize the resulting system of equations.

Plates on elastic foundations have been widely adopted by many researchers to model interaction between elastic media and plates for various engineering plate problems. However, the vibration analyses of FG plates on elastic foundations are limited and were performed based on the 2D theories [1,2,4,14]. This apparent void has thus formed the motivation of the present work. More recently, static analysis of simply supported FG beams [15] and rectangular plates [16] based on the elasticity theory were investigated using the state space method.

Here, the three-dimensional free of vibration analysis of FG plates on elastic foundations with some different boundary conditions are presented. FG plates with two opposite edges simply supported and arbitrary conditions at the other edges are considered. The material properties are assumed to be graded in the thickness direction and can vary according to power law distributions in terms of the volume fractions of the constituents, exponentially or any other formulations in this direction. A semi-analytical method, which makes use of a hybrid series solution and the dimensional differential quadrature method [17–20], is employed. Specially using DQM along the graded direction enables one to accurately and efficiently discretize the variable coefficient partial differential equations in this direction and implement the top and bottom surface boundary conditions of the plate, which the later includes the foundation effects. The accuracy and convergence of the present method are demonstrated through numerical results. A detailed parametric study is carried out to highlight the influences of thickness ratios, material property graded indexes, coefficients of elastic foundations and boundary conditions on the vibration frequencies of FG thick plates.

2. Governing equations

Consider a thick FG plate rested on two-parameter elastic foundations as shown in Fig. 1. A Cartesian coordinate system (x, y, z) is used to label the material point of the plate in the unstressed reference configuration. The plate is supported on elastic foundations at its lower surface. The Pasternak model is used to describe the reaction of the elastic foundation on the plate.

The material properties of the plate are assumed to vary continuously through the thickness of the plate. In this study, two different variation laws for the material properties are considered: power law and exponential distribution through the thickness. However, the formulation is so general that other variation laws of thickness coordinate can be easily implemented. Also, it is assumed that the Poisson's ratio ν is constant. Based on the power law distribution, the material elastic coefficients C_{ij} and the mass density ρ are assumed to be in terms of a power law distribution as follows,

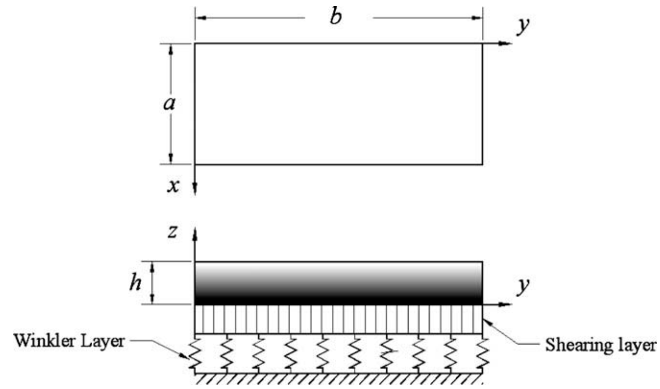

Fig. 1. FGM plate resting on elastic foundation.

$$C_{ij}(z) = C_{ij}^{M} + (C_{ij}^{C} - C_{ij}^{M})(z/h)^{p}, \quad \rho(z) = \rho^{M} + (\rho^{C} - \rho^{M})(z/h)^{p} \tag{1}$$

where superscripts M and C refer to the metal and ceramic constituents which denote the material properties of the bottom and top surface of the plate, respectively; p is the power law index or the material property graded index. On the other hands, for the exponential distribution, it is assumed that the material properties vary exponentially through the thickness of the plate as,

$$C_{ij}(z) = C_{ij}^{M} e^{(\gamma z/h)}, \quad \rho(z) = \rho^{M} e^{(\gamma z/h)} \tag{2}$$

where γ is the material property graded index.

Using the three-dimensional constitutive relations and the strain–displacement relations, the equations of motion in terms of displacement components for a linear elastic FG plate with infinitesimal deformations can be written as

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{66}\frac{\partial^2 u}{\partial y^2} + C'_{55}\frac{\partial u}{\partial z} + C_{55}\frac{\partial^2 u}{\partial z^2} + (C_{12}+C_{66})\frac{\partial^2 v}{\partial x \partial y} + C'_{55}\frac{\partial w}{\partial x} + (C_{13}+C_{55})\frac{\partial^2 w}{\partial x \partial z} = \rho\frac{\partial^2 u}{\partial t^2} \tag{3}$$

$$(C_{12}+C_{66})\frac{\partial^2 u}{\partial x \partial y} + C_{66}\frac{\partial^2 v}{\partial x^2} + C_{22}\frac{\partial^2 v}{\partial y^2} + C'_{44}\frac{\partial v}{\partial z} + C_{44}\frac{\partial^2 v}{\partial z^2} + C'_{44}\frac{\partial w}{\partial y} + (C_{23}+C_{44})\frac{\partial^2 w}{\partial y \partial z} = \rho\frac{\partial^2 v}{\partial t^2} \tag{4}$$

$$C'_{13}\frac{\partial u}{\partial x} + (C_{13}+C_{55})\frac{\partial^2 u}{\partial x \partial z} + C'_{23}\frac{\partial v}{\partial y} + (C_{23}+C_{44})\frac{\partial^2 v}{\partial y \partial z} + C_{55}\frac{\partial^2 w}{\partial x^2} + C_{44}\frac{\partial^2 w}{\partial y^2} + C'_{33}\frac{\partial w}{\partial z} + C_{33}\frac{\partial^2 w}{\partial z^2} = \rho\frac{\partial^2 w}{\partial t^2} \tag{5}$$

where u, v, and w are the displacement components along the x, y and z-axes; and $C'_{ij} = \frac{dC_{ij}}{dz}$. Eqs. (3 and 4) represent the in-plane equations of motion along the x and y-axes, respectively; and Eq. (5) is the transverse or out-of-plane equation of motion. The related boundary conditions are as follows:

At $z = 0, h$:

$$\sigma_{zx} = 0, \quad \sigma_{zy} = 0, \quad \sigma_{zz} = \begin{cases} k_w w - k_g\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) & \text{at } z = 0 \\ 0 & \text{at } z = h \end{cases} \tag{6}$$

where σ_{ij} are the components of stress tensor; k_w and k_g are the Winkler and shearing layer elastic coefficients of the foundations.

At $x = 0, a$:

$$\text{Either } u = 0 \text{ or } \sigma_{xx} = 0; \quad \text{either } v = 0 \text{ or } \sigma_{xy} = 0; \quad \text{either } w = 0 \text{ or } \sigma_{xz} = 0 \tag{7}$$

At $y = 0, b$:

$$\text{Either } u = 0 \text{ or } \sigma_{xy} = 0; \quad \text{either } v = 0 \text{ or } \sigma_{yy} = 0; \quad \text{either } w = 0 \text{ or } \sigma_{yz} = 0 \tag{8}$$