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Dynamic stiffness analysis of laminated composite beams using trigonometric shear deformation theory

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ABSTRACT

The exact dynamic stiffness matrix of a uniform laminated composite beam based on trigonometric shear deformation theory is developed in this paper. A refined laminated beam constitutive equation is derived that takes into account the breadth direction strains. The dynamic stiffness matrix is formulated directly in an exact sense by solving the governing differential equations of motion that describe the deformations of laminated beams according to the trigonometric shear deformation theory, which includes the sinusoidal variation of the axial displacement over the cross-section. The derived dynamic stiffness matrix is then used in conjunction with the Wittrick–Williams algorithm to compute the natural frequencies and mode shapes of the composite beams. The results obtained from the present theory are compared with those available in the literature wherever possible.

1. Introduction

The laminated composite beams are basic structural components when dealing with a variety of engineering structures such as airplane wings, helicopter blades and turbine blades as well as many others in the aerospace, mechanical, and civil industries. The great possibilities provided by composite materials can be used to alter or change favorably the response characteristics of structures. Due to the outstanding engineering properties, such as high strength-to-weight and stiffness-to-weight ratios, the composite beam structures are likely to play a significant role in the design of structures when the weight and strength are of primary consideration. The behavior of composite beam structures can also be effectively and efficiently tailored by changing the lay-up parameters. The analysis of composite beams is, of course, significantly more difficult than that of metallic counterparts. An important element in the dynamic analysis of composite beams is the computation of their natural frequencies and mode shapes. This is important as the composite beam structures often operate in complex environmental conditions and are frequently exposed to a variety of dynamic excitations. The researches pertain to the vibration analysis of composite beams have undergone rapid growth over the past few decades and are still growing. A large number of investigators address the problem of vibration analysis of laminated composite beams.

The classical laminated beam theory, based on Euler–Bernoulli hypothesis, is inaccurate for reasonably thick laminated beams and/or for highly anisotropic composite beams. The inaccuracy is due to neglecting the transverse shear and normal strains in the laminate. Transverse shear deformation is a major issue in the analysis of composite beams. A lot of work has been done to estimate it correctly and efficiently for design purposes. In order to account for the effect of low ratio of transverse shear modulus to the in-plane modulus, the first-order shear deformation theory (FSDT) in which the transverse shear strain is assumed to be constant in the beam depth direction has been developed.

Teh and Huang [1] presented two finite element models based on FSDT for the free vibration analysis of fixed-free beams of general orthotropy. Chen and Yang [2] reported the free vibration analysis of symmetrically laminated beams based on FSDT using the finite element approach. Chandrashekhara et al. [3] obtained the exact modal solutions for symmetrically laminated beams based on FSDT including rotary inertia. Krishnaswamy et al. [4] used Hamilton's principle to formulate the dynamic equations governing the free vibration of laminated composite beams. The influences of transverse shear deformation and rotary inertia were included, and analytical solutions for unsymmetrically laminated beams were obtained by applying Lagrange multipliers method. Abramovich and Livshits [5] reported the free vibration analysis of non-symmetric cross-ply laminated beams based on FSDT. Nabi and Ganesan [6] examined bi-axial bending, axial and torsional vibrations using the finite element method and FSDT. Eisenberger et al. [7] obtained the analytical solutions for generally laminated beams based on FSDT including rotary inertia. Teboub and Hajela [8] adopted the symbolic computation technique to analyze the free vibration of generally layered composite beam on the basis of FSDT. Banerjee and Williams [9] presented an exact dynamic

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stiffness matrix for a composite beam with the effects of shear deformation, rotary inertia and coupling between the bending and torsional deformations included. Yildirim [10] used the stiffness method for the solution of in-plane free vibration problem of symmetric cross-ply laminated beams with the effects of rotary inertia, axial and transverse shear deformations included by FSDT. Chakraborty et al. [11] presented a new refined locking-free first-order shear deformable finite element and demonstrated its utility in solving free vibration and wave propagation problems in laminated beam structures with symmetric as well as asymmetric ply-stacking. Mahapatra and Gopalakrishnan [12] presented a spectral finite element model for analysis of axial-flexural-shear coupled wave propagation in thick laminated composite beams and derived an exact dynamic stiffness matrix. Ruotolo [13] developed a spectral element for anisotropic, laminated composite beams. The axial-bending coupled equations of motion were derived under the assumptions of FSDT. Goyal and Kapania [14] proposed a 21 degree-of-freedom element to study the response of unsymmetrically laminated composite structures subject to both static and dynamic loads based on FSDT.

In FSDT a shear correction factor has to be incorporated to adjust the transverse shear stiffness. The accuracy of solutions of FSDT will be dependent on the shear correction factor. It has been shown that FSDT is sometimes inadequate to predict the accurate solutions of laminated composite beams. Several refined one-dimensional higher-order shear deformation theories which take into account the effects of transverse shear and normal stresses and rotary inertia have been proposed to analyze the dynamic characteristics of laminated composite beams.

Krishna Murty [15] developed the higher-order shear deformation theories based on the elementary theory of beam and applied these theories to the vibration analysis of laminated beams. Bhimaraddi and Chandrashekhara [16] modeled the composite beams by a systematic reduction of the constitutive relations of the three-dimensional anisotropic body and obtained the basic equations of the beam theory based on the parabolic shear deformation theory. Soldatos and Elishakoff [17] developed a third-order shear deformation theory for static and dynamic analysis of an orthotropic beam incorporating the effects of transverse shear and normal deformations. Chandrashekhara and Bangera [18] studied the free vibration characteristics of laminated beams by the finite element method using a third-order shear deformation theory. Singh and Abdelnassar [19] worked out the free and forced vibration of symmetric cross-ply composite beams based on third-order shear deformation theory. Khdeir and Reddy [20] developed the analytical solutions of refined beam theories to study the free vibration behavior of cross-ply rectangular beams with arbitrary boundary conditions. Marur and Kant [21] compared higher-order refined theories on the transient response of laminated composite beams, showing the effects of shear deformation and rotary inertia. Kant et al. [22] presented an analytical method for the dynamic analysis of laminated beams using higher-order refined shear deformation theory. Song and Waas [23] studied the buckling and free vibration of stepped laminated beams employing a simple higher-order shear deformation theory in which a cubic distribution of the displacement field through the beam thickness was assumed. Shimpi and Ghugal [24] presented a layerwise trigonometric shear deformation theory for the static analysis of two-layered cross-ply laminated beams. Shimpi and Ainapure [25] studied the free vibration of two-layered laminated cross-ply beams using the variationally consistent layerwise trigonometric shear deformation theory. Shi and Lam [26] presented a new finite element formulation for the free vibration analysis of composite beams based on the third-order beam theory. Shimpi and Ainapure [27] presented a simple one-dimensional beam finite element, based on variationally consistent layerwise trigonometric shear deformation theory. Shimpi and Ghugal [28] proposed a new layerwise trigonometric shear deformation theory for the flexural analysis of two-layered cross-ply laminated beams. The sinusoidal function in terms of thickness coordinate is used in the displacement field to account for shear deformation. Ghugal and Shimpi [29] presented a review of displacement and stress based refined theories for isotropic and anisotropic laminated beams and discussed various equivalent single layer and layerwise theories for laminated beams. Rao et al. [30] proposed a higher-order mixed theory for determining the natural frequencies of a diversity of laminated simply-supported beams. Matsunaga [31] studied the natural frequencies, buckling stresses and interlaminar stresses of composite beams with simply-supported edges using a global higher-order theory. Arya et al. [32] developed a zigzag model for symmetric laminated beam in which a sine term was used to represent the nonlinear displacement field across the thickness as compared to a third-order polynomial term in conventional theories. Zero transverse shear stress boundary condition at the top and bottom of the beam were satisfied. Chen et al. [33] presented a new approach combining the state space method and the differential quadrature method for free vibrating laminated beams based on two-dimensional theory of elasticity. Chen et al. [34] presented a new method of state space based differential quadrature for free vibration of generally laminated beams. Murthy et al. [35] developed a refined 2-node, 4 DOF/node beam element based on higher-order shear deformation theory for axial-flexural-shear coupled deformation in asymmetrically stacked composite beams. Aydogdu [36] carried out the vibration analysis of cross-ply laminated beams with different sets of boundary conditions by use of the Ritz method. The analysis was based on a three-degree-of-freedom shear deformable beam theory. Aydogdu [37] investigated the free vibration of angle-ply laminated beams subjected to different sets of boundary conditions by applying the Ritz method. The analysis was based on a three-degrees-of-freedom shear deformable beam theory. Subramanian [38] performed the free vibration analysis of laminated composite beams using two higher-order displacement based shear deformation theories. Both theories assumed a quintic and quartic variation of in-plane and transverse displacements in the thickness coordinates of the beams, respectively. Tahani [39] developed a new layerwise laminated beam theory for generally laminated beams. Analytical solutions for static bending and free vibration were obtained for the bending and free vibration of cross-ply and angle-ply laminated beams by using the state space method.

Although there are many references available on free vibration analysis of laminated composite beams, most of which deal with cross-ply composite beams and the dynamic behavior of generally laminated composite beams are not well-documented. To the author's knowledge, there are no reported investigations on free vibration of arbitrary lay-up composite beams based on trigonometric shear deformation theory documented in the literature. This problem has received limited attention. In this paper a dynamic stiffness formulation is introduced in order to investigate the free vibration characteristics of the generally laminated composite beams. It is well-known that the dynamic stiffness method relies on the exact solutions of the governing differential equations of motion, therefore it can be classified as an exact method, the term exact referring of course to the range of validity of the adopted beam theory. Compared to the conventional finite element or other approximate methods, the dynamic stiffness method becomes very useful when higher frequencies and/or better accuracies of the results are required.

The governing equations of motion of the general ply-stacking composite beams are derived via Hamilton's principle, where the Poisson effect and trigonometric shear deformation theory are considered. The trigonometric shear deformation theory incorporates the sinusoidal variation of in-plane displacement through the