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Stiffness loss of laminated composite plates with distributed damage by the modified local Green's function method

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Abstract

The modified local Green's function method (MLGFM) is an integral method which has been applied in many structural and continuum mechanics problems. It uses the finite element method as an auxiliary tool to automatically develop the Green's functions projections, which correspond to the fundamental solutions of the boundary element method. The same numerical approach techniques of the FEM and the BEM are employed. The advantages of this method are high convergence and precision, especially for tractions and fluxes on the boundaries. The aim of this paper is to show the MLGFM capability to solve laminated composite plates with generalized transverse cracks. The damage effect is considered as an additional load vector and no modification is necessary in the stiffness matrix of the problem. Some examples are presented and the results are compared within the ones chosen from the referred literature.

Keywords: Damage; Transverse cracks; Laminated composite plates; Internal state variable; Modified local Green's function method

1. Introduction

The damage evolution of the composite laminated plates is, in general, a quite complex process. According to Allen et al. [1,2] and Tay et al. [3], transversal cracks in off-axis plies usually constitute the first internal damage of unidirectional fiber reinforced composites. The loss of structural integrity in composite structures is related to the form and wideness of damage and also to the loading mode. The continuum damage mechanics has demonstrated good approximation and a suitable methodology for the case of generalized distribution of cracks [4].

The stiffness properties of composite laminates depend on the generalized cracks in the polymeric matrix, as shown by Talreja [5]. Numerous models of composite structures with distributed and localized damage are found in the literature and some of them are referred here [5–14]. The main objectives of these methods are to determine the constitutive behavior of layered composites involving cracks in the matrix; and to evaluate the crack opening, the stiffness reduction or the damage evolution.

Simplified models were developed for orthogonally cracked laminates, which are used to evaluate the changes in stiffness caused by crosswise crack systems [15–24].

By using the concept of internal variable [1,2,18], it is possible to consider the damage effect as an additional load vector, which depends on the opening and the density of cracks [20,21]. It is supposed that the cracks are always transversal to the fibers and that they occur in the polymeric matrix. The cracked laminated plates depend on the ply orientation and the laminate stack. In such a way, the additional load vector is superimposed to the external load vector and the constitutive matrix has no change during the damage evolution.

Since the objective of this work is to apply an integral numerical method to solve angle ply symmetric laminates

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with generalized transversal cracks, simplified models are used here to simulate the damage effect.

2. Damage model for symmetric angle ply laminates

In this work, the first order shear deformation laminated plate theory (FSDT) [25] is used to formulate the equations for composite laminates with transverse cracks. The laminated plate is represented by its middle surface and, considering $\{N\}$ and $\{M\}$ as the resultants forces and moments vectors, σ_x, σ_y, τ_{xy}, the conventional stress on the plate in some point and t the laminate thickness, it is possible to write:

$$\{N\} = \int_{-t/2}^{t/2} \{\, \sigma_x \quad \sigma_y \quad \tau_{xy} \,\}^{\mathrm{T}} \mathrm{d}z \tag{1}$$

$$\{M\} = \int_{-t/2}^{t/2} \{\, \sigma_x \quad \sigma_y \quad \tau_{xy} \,\}^{\mathrm{T}} z \mathrm{d}z \tag{2}$$

The continuum damage hypothesis involves internal state variables (ISVs), which are strain-like variables containing kinematics features of the distributed transverse cracks or damage [1,2,18]. The ISVs are employed for establishing constitutive relations in the layer material coordinates. Some variables, such as the crack opening and sliding displacements, can be approximated by simple functions and they are relevant for the evaluation of ISVs.

For non-cracked plates, the vectors $\{N\}$ and $\{M\}$ are related to the strain and mid-plane curvature vectors, $\{\varepsilon_0\}$ and $\{\kappa_0\}$, by the extensional, flexural, and coupling extensional–flexural effective stiffness matrices, respectively, $[A]$, $[D]$ and $[B]$. These matrices can be built in a conventional way as in the single layer models [25]. However, for cracked symmetric plates, it must be superimposed the membrane $\{D_N\}$ and bending $\{D_M\}$ damage vectors [3,16–21]. In this case, the complete equation system is

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \kappa_0 \end{Bmatrix} + \begin{Bmatrix} D_N \\ D_M \end{Bmatrix} \tag{3}$$

The ISVs are employed to build the damage load vectors, $\{D_N\}$ and $\{D_M\}$ of (3), which are developed by taking the total number of damage types for each layer and the layer contribution for the laminated plate. The density, opening and orientation of transverse cracks affect the mechanical behavior of laminated plate. The effects on the plate of the damage load vectors are then similar to the stiffness loss of cracked plate.

Considering a conventional approach for the structure by finite elements, the weak formulation for the plate equilibrium equations results in the following algebraic equations system:

$$\begin{bmatrix} K_{11} & K_{12} & K_{16} \\ K_{21} & K_{22} & K_{26} \\ K_{61} & K_{62} & K_{66} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \begin{Bmatrix} F_1^{\mathrm{a}} \\ F_2^{\mathrm{a}} \\ F_6^{\mathrm{a}} \end{Bmatrix} + \begin{Bmatrix} F_1^{\mathrm{d}} \\ F_2^{\mathrm{d}} \\ F_6^{\mathrm{d}} \end{Bmatrix} \tag{4}$$

where $[K_{ij}]$ are the local stiffness matrix, $\{u_x, u_y, u_z\}^{\mathrm{T}}$ is the element displacement vector, and $\{F^{\mathrm{a}}\}$ and $\{F^{\mathrm{d}}\}$ are the applied load vector and the element damage load vector, respectively. Taking $p_x^{\mathrm{a}}, p_y^{\mathrm{a}}, p_z^{\mathrm{a}}$ and $p_x^{\mathrm{d}}, p_y^{\mathrm{d}}, p_z^{\mathrm{d}}$ as the distributed external forces and equivalent damage forces on the three directions, and ψ the interpolation functions, one is possible to write:

$$F_1^{\mathrm{a}} = \int_{\Omega^e} p_x^{\mathrm{a}} \psi_1 \,\mathrm{d}x\,\mathrm{d}y \tag{5}$$

$$F_2^{\mathrm{a}} = \int_{\Omega^e} p_y^{\mathrm{a}} \psi_2 \,\mathrm{d}x\,\mathrm{d}y \tag{6}$$

$$F_6^{\mathrm{a}} = \int_{\Omega^e} p_z^{\mathrm{a}} \psi_6 \,\mathrm{d}x\,\mathrm{d}y \tag{7}$$

$$F_1^{\mathrm{d}} = \int_{\Omega^e} p_x^{\mathrm{d}} \psi_1 \,\mathrm{d}x\,\mathrm{d}y \tag{8}$$

$$F_2^{\mathrm{d}} = \int_{\Omega^e} p_y^{\mathrm{d}} \psi_2 \,\mathrm{d}x\,\mathrm{d}y \tag{9}$$

$$F_6^{\mathrm{d}} = \int_{\Omega^e} p_z^{\mathrm{d}} \psi_6 \,\mathrm{d}x\,\mathrm{d}y \tag{10}$$

Considering $\overline{Q}_{ij}$ $(i, j = 1, 2, 6)$ as the transformed reduced stiffness matrix and z_{k-1} and z_k the mid-plane distances of the laminate until the bottom and the top surfaces of a generic lamina k, the components $p_x^{\mathrm{d}}, p_y^{\mathrm{d}}, p_z^{\mathrm{d}}$ are related to the ISVs, α_{xx}, α_{yy}, and α_{xy}, and can be defined as following.

$$p_x^{\mathrm{d}} = \left[\frac{\partial}{\partial x} \sum_{k=1}^{N} \{\overline{Q}_{11}\alpha_{xx} + \overline{Q}_{12}\alpha_{yy} + \overline{Q}_{16}\alpha_{xy}\}_k + \frac{\partial}{\partial y} \sum_{k=1}^{N} \{\overline{Q}_{16}\alpha_{xx} + \overline{Q}_{26}\alpha_{yy} + \overline{Q}_{66}\alpha_{xy}\}_k \right] (z_k - z_{k-1}) \tag{11}$$

$$p_y^{\mathrm{d}} = \left[\frac{\partial}{\partial x} \sum_{k=1}^{N} \{\overline{Q}_{16}\alpha_{xx} + \overline{Q}_{26}\alpha_{yy} + \overline{Q}_{66}\alpha_{xy}\}_k + \frac{\partial}{\partial y} \sum_{k=1}^{N} \{\overline{Q}_{12}\alpha_{xx} + \overline{Q}_{22}\alpha_{yy} + \overline{Q}_{26}\alpha_{xy}\}_k \right] (z_k - z_{k-1}) \tag{12}$$

$$p_z^{\mathrm{d}} = \left[\frac{1}{2} \frac{\partial^2}{\partial x^2} \sum_{k=1}^{N} \{\overline{Q}_{11}\alpha_{xx} + \overline{Q}_{12}\alpha_{yy} + \overline{Q}_{16}\alpha_{xy}\}_k + \frac{\partial^2}{\partial x \partial y} \sum_{k=1}^{N} \{\overline{Q}_{16}\alpha_{xx} + \overline{Q}_{26}\alpha_{yy} + \overline{Q}_{66}\alpha_{xy}\}_k + \frac{1}{2} \frac{\partial^2}{\partial y^2} \sum_{k=1}^{N} \{\overline{Q}_{12}\alpha_{xx} + \overline{Q}_{22}\alpha_{yy} + \overline{Q}_{26}\alpha_{xy}\}_k \right] (z_k^2 - z_{k-1}^2) \tag{13}$$

Once the ISVs are known, by replacing (11)–(13) to (8)–(10) one obtains the damage load vector. This vector is added to the nodal forces of the structure (4) to complete the global equation system.

The method shown here is at great length in the papers of Talreja et al. [5,18,19] and Tay et al. [3,20,21]. It can be implemented without major changes in a conventional FEM code. The present paper uses the same methodology