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A constitutive model for fiber-reinforced polymer plies accounting for plasticity and brittle damage including softening – Implementation for implicit FEM

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ABSTRACT

A constitutive ply model is developed to allow for the simulation of laminated structures made of fiber-reinforced polymers. The model accounts for stiffness degradation accompanied by strain hardening (i.e. distributed brittle damage), unrecoverable strain accumulation (i.e. multi-surface plasticity), and stiffness degradation accompanied by strain softening (i.e. localized brittle damage). Owing to the characteristics of such ply materials the model handles these effects and their evolutions in an anisotropic manner.

The constitutive model is briefly reviewed with respect to distributed damage and plasticity, and the extension to incorporate softening is given in detail. The implementation as material routine for an implicit FEM package and the computation of the material Jacobian matrix is presented. Additional measures are adopted for improving/achieving convergence, as being required for the treatment of material softening within numerical solution schemes.

The application of the constitutive model for structural analysis is demonstrated by presenting simulation results from Open Hole Tension configurations. The predictions include the overall non-linear response until structural failure and the local distribution and evolution of the mechanisms incorporated in the constitutive ply model. Discussion of these results and comparison to literature show that crucial intra-ply mechanisms are captured well by the proposed constitutive model.

1. Introduction

Laminated composites made of unidirectional (UD), continuous fiber reinforced polymers (FRPs) have become very popular in the past decades. The increasing usage primarily originates from their weight saving potential, which in turn comes from their high weight specific stiffness and their high weight specific strength. To further exploit the advantages of FRP laminates, reliable methods are required to predict their mechanical response beyond elasticity. Damage, plasticity, and material softening should be handled in a highly direction dependent manner. Applicability of these methods within the framework of the Finite Element Method (FEM) is desired in order to allow for the analyses of entire components and parts up to structural failure.

For modeling and simulation of FRP laminates within FEM, the following approach is often utilized. First of all, the UD plies are modeled as smeared out (homogenized) materials, i.e. transverse isotropy is assumed initially with the plane perpendicular to the fibers being the plane of material isotropy. Perfect interfaces between the plies and plane stress states within the plies are assumed. Finally, the transition from the ply to the laminate length scale is performed by FEM via the intrinsic formulation of elements that allow for layered section definitions. With the outlined approach, it is left to formulate and implement a constitutive model for the smeared out ply material, preferably taking into account non-linear ply behavior, too.

In the past decades and years, many models describing the smeared out ply material behavior have been proposed in the literature. Many of these models incorporate damage mechanics; the variety ranges from rather straightforward methods where the ply stiffness properties are progressively degraded using degradation functions, see e.g. [1], or uniaxial considerations (e.g. [2]) to sophisticated continuum damage models as proposed by a group from Cachan [3–5] or by other authors, see e.g. [6–8], and [9] which has also been combined with viscoelasticity [10]. The microcrack characteristics is taken into account in [11] in an elasto-damage model. Several models focus on strain softening behavior, i.e. they model FRP composites as anisotropic elastic–brittle continua, see e.g. [12–14]. Models with special emphasis on compressive failure have been proposed as well, e.g. [15,16]. Other models focus on the pronounced in-elasticity which is observed under shear dominated loading, see e.g. [17]. Models accounting for stiffness degradation as well as unrecoverable strain accumulation have been proposed e.g. in [18,19].

Based on former work [20–22], a constitutive model for the smeared out ply material has been proposed by the authors

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[23,24] which accounts for non-linearities induced by anisotropic stiffness degradation accompanied by strain hardening as well as non-linearities induced by the accumulation of unrecoverable strains. This model and its basic features are reviewed in the first part of Section 2. In the second part of Section 2 the constitutive model is extended to take into account anisotropic stiffness degradation accompanied by strain softening. Here, it shall be pointed out that the proposed model is not designed with respect to particular loading scenarios and the accompanying failure patterns but can handle arbitrary loading scenarios including non-proportional as well as un-and reloading. In Section 3 the implementation of the complete constitutive model within the framework of implicit FEM is discussed. In this context, the algorithmic procedure is presented which is utilized to solve for the unknown stress and material states induced by a given strain increment. Furthermore, the derivation of the material Jacobian matrix is treated since its consistent computation is a prerequisite for quadratic convergence of the global Newton–Raphson iteration of the FEM solution. As the constitutive model is implemented as user defined material routine (UMAT) for the commercial FEM package Abaqus/Standard 6.9 (*Dassault-Systèmes Simulia Corp., Providence, RI, USA*), it is readily applicable in non-linear FEM simulations. To demonstrate this applicability, results from non-linear FEM simulations of layered composite structures are presented in Section 4.

2. The constitutive model

The proposed constitutive model describes the behavior of the smeared out ply material, but only plies are considered which are embedded in a laminate. Furthermore, the development of the model is done under small strain and plane stress assumption as typically applicable in analyses of laminated, thin-walled components and structures.

During loading of a laminate various phenomena may occur within the plies leading to a pronounced non-linear response. Within the constitutive model not all the individual phenomena are distinguished but their effects are modeled in an averaged sense. Nevertheless, two major types of effects are distinguished due to their opposed characteristics. These two types are, first, stiffness degradation and, second, the accumulation of unrecoverable strains. Stiffness degradation is incorporated in the present constitutive model via brittle continuum damage mechanics, i.e. via elasto-damage, whereas the evolution of the unrecoverable strains is described by a plasticity model. Accordingly, unrecoverable strains are referred to as plastic strains in the following.

The two outlined model parts, i.e. stiffness degradation and plastic strain accumulation, are defined with respect to the stress and strain history by separate, phenomenological, incremental formulations. For the purpose of their combination in a single elasto–plasto-damage material model, the elasto-damage model concerning stiffness degradation is enriched by the model which treats the evolution of plastic strains. Accordingly, after appropriate integration of the loading history, the mechanical strain is composed of an elastic as well as a plastic contribution and reads

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{(\mathrm{el})} + \boldsymbol{\varepsilon}^{(\mathrm{pl})} = \boldsymbol{C}\boldsymbol{\sigma} + \boldsymbol{\varepsilon}^{(\mathrm{pl})}. \tag{1}$$

Here $\boldsymbol{\varepsilon}$ is the vector of the mechanical strain components, i.e. the total strains minus thermal strains, $\boldsymbol{\varepsilon}^{(\mathrm{el})}$ is the vector of the elastic strain components, $\boldsymbol{\varepsilon}^{(\mathrm{pl})}$ is the vector of the accumulated plastic strain components, $\boldsymbol{C}$ is the current compliance matrix of the (possibly damaged) ply material, and $\boldsymbol{\sigma}$ is the vector of stress components. In Eq. (1), both the compliance matrix as well as the vector of plastic strain components contribute to the non-linearity of the ply behavior.

2.1. Stiffness degradation accompanied by strain hardening

Stiffness degradation accompanied by strain hardening is attributed to microscopic brittle matrix cracking and fiber matrix debonding, i.e. evenly distributed matrix dominated phenomena. The respective model concept is denoted as 'distributed brittle damage'. Since the effects of such phenomena are incorporated in the present model by means of continuum damage mechanics, they are reflected only by a change in the compliance matrix, $\boldsymbol{C}$, of the smeared out ply material. In order to cope with the challenging task of estimating the compliance matrix, $\boldsymbol{C}$, in a physically meaningful way, the model is based on an approach in which the damaged ply material is featured by an appropriate fictitious material.

Before any loading is applied to the laminate, the fictitious material consists of the virgin but homogenized ply material. As soon as damage accumulates spheroidal, oblate inhomogeneities are successively 'added' to the fictitious material. This means that the volume fraction of the inhomogeneities is zero in the undamaged state but increases with damage accumulation. Note that the volume fraction cannot decrease even if the ply is unloaded. The orientation of inhomogeneities depends on the loading condition under which they are formed and does not change during further loading. In order to model the effect of distributed matrix dominated phenomena as being treated by the present model part, three different orientations of the inhomogeneities are considered leading to three populations $p = 2, 3, 4$ of inhomogeneities. Each population is described by its volume fraction, $\xi^{(p)}$. The three different populations are designed to incorporate different matrix damage modes as observed in experimental testing; the total amount of matrix damage is defined by

$$\xi^{(\mathrm{m})} = \sum_{p=2}^{4} \xi^{(p)}. \tag{2}$$

In contrast to the orientation, the stiffness of the inhomogeneities depends on the actually prevailing loading conditions. Under tensile loading conditions, the inhomogeneities possess zero stiffness (voids), i.e. they model the anisotropic effect of open cracks. Under compressive loading conditions, the inhomogeneities possess non-zero stiffness, i.e. they model the anisotropic effect of closed cracks.

This fictitious material allows to apply a method borrowed from micromechanics and to derive 'damage matrices', $\boldsymbol{D}^{(p)}$, such that the compliance matrix of the degraded material can be written as

$$\boldsymbol{C} = \left(\boldsymbol{I} + \sum_{p} \boldsymbol{D}^{(p)}\right)\boldsymbol{C}^{(0)}, \tag{3}$$

where $\boldsymbol{I}$ stands for the unit matrix and $\boldsymbol{C}^{(0)}$ is the compliance matrix of the undamaged ply material. For the case of inhomogeneities with zero stiffness (voids), the damage matrix reads

$$\mathbf{D}^{(p)} = \frac{\xi^{(p)}}{1 - \sum_p \xi^{(p)}} \left[\mathbf{I} - \mathbf{S}^{(p)}\right]^{-1}, \tag{4}$$

where $\xi^{(p)}$ stands for the volume fraction and $\mathbf{S}^{(p)}$ is Eshelby tensor of the considered population p. The outlined approach is advantageous as the damage matrices, $\boldsymbol{D}^{(p)}$, model the anisotropic effect of brittle damage, although they primarily depend only on scalar, and thus handy, damage variables $\xi^{(p)}$, see [20,21,24]. With Eqs. (3) and (4) it remains to relate the damage variables, $\xi^{(p)}$, to the ply loading history. To this end damage evolution equations in the form of

$$\xi^{(\mathrm{m})} = k_{\mathrm{d}}^{(\mathrm{m})}\left(f_{\mathrm{E}}^{(\mathrm{m})} - 1\right)^2 \tag{5}$$

are introduced with the evolution parameter, $k_{\mathrm{d}}^{(\mathrm{m})}$, and the factor of exertion, $f_{\mathrm{E}}^{(\mathrm{m})}$, which is a measure for the damage relevant load state