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Nonlinear free and forced vibration behavior of functionally graded plate with piezoelectric layers in thermal environment

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ABSTRACT

In the present study, finite element formulation based on higher order shear deformation plate theory is developed to analyze nonlinear natural frequencies, time and frequency responses of functionally graded plate with surface-bonded piezoelectric layers under thermal, electrical and mechanical loads. The von Karman nonlinear strain–displacement relationship is used to account for the large deflection of the plate. The material properties of functionally graded material (FGM) are assumed temperature-dependent. The temperature field has uniform distribution over the plate surface and varies in the thickness direction. The considered electric field only has non-zero-valued component E_z. Numerical results are presented to study effects of FGM volume fraction exponent, applied voltage in piezoelectric layers, thermal load and vibration amplitude on nonlinear natural frequencies and time response of FGM plate with integrated piezoelectric layers. In addition, nonlinear frequency response diagrams of the plate are presented and effects of different parameters such as FGM volume fraction exponent, temperature gradient, and piezoelectric voltage are investigated.

1. Introduction

Functionally graded materials (FGMs) have received considerable attention in many engineering applications since they were first reported in 1980s. FGMs are composite materials, microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. This is achieved by gradually varying the volume fraction of the constituent materials. FGMs are usually made from a mixture of metals and ceramics using powder metallurgy techniques. The ceramic material provides the high-temperature resistance due to its low thermal conductivity, while the ductile metal constituent prevents fracture due to its greater toughness. Unlike fiber–matrix composites, in which cracking and debonding may occur at high temperatures due to the material property mismatch at the interface of two discrete materials, FGMs have the advantage of being capable of withstanding severe high temperature while maintaining structural integrity. Birman and Byrd [1] presented a review of the principal developments in FGMs with an emphasis on the work published since 2000. Diverse areas relevant to various aspects of theory and applications of FGM including homogenization of particulate FGM, heat transfer issues, stress, stability and dynamic analyses, testing, manufacturing and design, applications, and fracture are reflected in this paper.

FGM plates are often susceptible to failure from large deflections, or excessive stresses that are induced by large temperature gradients and/or mechanical loads. Hence, it is of prime importance to understand the large amplitude (geometrically nonlinear) vibration behavior of FGM plate structures to ensure more accurate and reliable structural analysis and design. In recent years, the dynamics of FGM plates have attracted increasing research effort, most of which has been devoted to linear vibration and dynamic response. However, investigations on the large amplitude vibration and transient response of FGM plates are limited in number.

Based on the first order shear deformation plate theory (FSDT), Praveen and Reddy [2] analyzed geometrically nonlinear static and dynamic response of functionally graded plates subjected to transverse mechanical loads and temperature distribution by using finite element method. Reddy [3] presented the Navier solutions for linear bending of simply supported rectangular FGM plates and developed the finite element models based on the FSDT for nonlinear static and dynamic response of FGM plates under thermo-mechanical loads. Shen [4] employed a mixed Galerkin-perturbation technique to determine the nonlinear bending response of FGM rectangular plates subjected to transverse mechanical load in thermal environments. Based on improved perturbation technique, Huang and Shen [5] studied the nonlinear frequencies and dynamic response of functionally graded plates in thermal environments. Chen [6] investigated the nonlinear natural frequencies of functionally graded plates with general state of non-uniform initial stresses. The finite element formulation, based on

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the FSDT, has been developed by Sundararajan et al. [7] to analyze the large amplitude free flexural vibrations behavior of rectangular and skew FGM plates subjected to thermo-mechanical environments. An analytical solution based on classical plate theory, presented by Woo et al. [8] to analysis the nonlinear free vibration behavior of functionally graded plates in thermal environments. Park and Kim [9] developed the finite element equations based on the FSDT to analyze nonlinear vibration and post buckling behavior of functionally graded plates under thermal loads.

Vibration control of plate structures using smart materials such as piezoelectrics, shape memory alloys, and rheological fluids are increasingly important in practical applications. In recent decades, many studies have been carried out on the modeling and controlling of the vibration behavior of isotropic and composite plates by means of piezoelectric sensors and actuators. For example, several variational statements for the vibration analysis of multilayered piezoelectric plates and shells are presented by Carrera et al. [10]. With the increasing usage of FGMs, active control and vibration behavior analysis of FGM plates that are integrated with piezoelectric materials has becoming a subject of great interest, e.g. Liew et al. [11] presented an efficient finite element formulation based on FSDT for the active control of FGM plates with integrated piezoelectric sensor/actuator layers subjected to a thermal gradient. Investigations on the large amplitude vibration and transient response of FGM plate with integrated piezoelectric layers are limited in number.

Yang et al. [12] investigated nonlinear natural frequencies of FGM plate with surface-bonded piezoelectric layers subjected to uniform temperature change, in-plane forces and applied piezoelectric voltage. They presented a semi-analytical method based on one-dimensional differential quadrature and Galerkin technique using higher order shear deformation plate theory (HSDT). In their analysis, however, the heat conduction and/or temperature-dependent material properties are not accounted for. In addition, Yang et al. [13] investigated the nonlinear bending behavior of functionally graded plates that are bonded with piezoelectric layers and subjected to transverse loads and a temperature gradient based on HSDT. They employed one-dimensional differential quadrature and Galerkin technique in their analysis. Huang and Shen [14] studied the nonlinear frequencies and dynamic response of FGM plate with surface-bonded piezoelectric layers in thermal environments using improved perturbation technique based on HSDT. In this study, the heat conduction and temperature-dependent material properties were considered. Shen [15] presented nonlinear bending response of simply supported FGM plate without or with piezoelectric layers subjected to thermal and electrical loads. Heat conduction and temperature-dependent material properties were both taken into account. A two-step perturbation technique based on HSDT was employed in this study. Panda and Ray [16] presented a nonlinear static finite element analysis based on FSDT for simply supported functionally graded plates integrated with the patches of piezoelectric fiber reinforced composite material in the presence/absence of the thermal environment. Yiqi and Yiming [17] have studied the nonlinear dynamic response and active vibration control of the piezoelectric functionally graded plate based on HSDT and elastic piezoelectric theory using finite difference method. However, they did not consider thermal load. In addition, they did not investigate nonlinear natural frequency and nonlinear frequency response of the piezoelectric functionally graded plate.

To the best of the author's knowledge, the finite element formulation to investigate geometrically nonlinear vibration behavior (nonlinear natural frequency, time and frequency response) of FGM plate with integrated piezoelectric layers has not been presented in any literature. In the present study, finite element formulation based on HSDT is developed to analyze nonlinear natural frequencies and dynamic response of FGM plate with surface-bonded piezoelectric layers under thermal, electrical, and mechanical loads. The geometric nonlinearity using von Karman's assumption is introduced. The material properties of FGM are assumed temperature-dependent and are graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. The temperature field is assumed constant in the plane and varies only in the thickness direction of the plate and the electric field considered only has non-zero-valued component E_z.

2. Analytical model and general comments

Consider an FGM plate with surface-bonded piezoelectric layers. The length, width and total thickness of the hybrid laminated plate are a, b and h. The thickness of the FGM layer is h_f, while the thickness of each piezoelectric layer is h_p. The coordinate system to analyze vibration behavior of plate is shown in Fig. 1. The origin of this coordinate system is considered in the mid plane of the plate.

It assumed that the top surface of FGM layer ($z = h_f/2$) is ceramic-rich and its bottom surface ($z = -h_f/2$) is metal-rich. The effective material properties of FGM layer are assumed temperature-dependent and are graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents, as follows

$$P(z,T) = (P_c - P_m)\left(\frac{2z + h_f}{2h_f}\right)^n + P_m \tag{1}$$

where P denotes a generic material property, P_c and P_m are the corresponding values of the ceramic and metal constituents and n is the volume fraction exponent that is the positive real value.

The constituent material properties are considered temperature-dependent as follows [18]

$$P = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{2}$$

where P_{-1}, P_0, P_1, P_2 and P_3 are the coefficients of temperature T (K) and are unique to each constituent.

The temperature variation is assumed to occur in the thickness direction of laminated plate and the temperature field is considered constant in the xy-plane. Also, it is assumed that the changes in thermal conditions happen very fast and so there is no thermal dissipation in the processes (adiabatic processes). In such a case, the temperature distribution along the thickness can be obtained by solving a steady-state heat transfer equation

$$-\frac{d}{dz}\left(K(z)\frac{dT}{dz}\right) = 0 \tag{3a}$$

where

$$K(z) = \begin{cases} k_p & h_f/2 < z < h_p + h_f/2 \\ k_f(z) & -h_f/2 < z < h_f/2 \\ k_p & -h_p - h_f/2 < z < -h_f/2 \end{cases} \tag{3b}$$

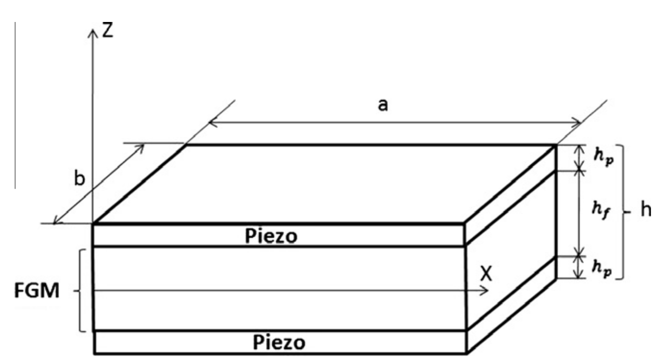

Fig. 1. The coordinate system considered to analyze vibration behavior of the plate.