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Variational-asymptotic modeling of the thermoelastic behavior of composite beams

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ABSTRACT

This paper deals with steady state thermoelastic problems in composite beam structure by using variational-asymptotic method. First, the original three-dimensional heat conduction problem is reduced to be a two-dimensional thermal cross-sectional analysis along with an optional one-dimensional heat conduction analysis. The one-dimensional heat conduction analysis exists only if the temperature is not prescribed at any point of the cross-section along the span except the end surfaces. Then we reduce the one-way coupled, three-dimensional thermoelasticity problem into a two-dimensional, one-way coupled thermoelastic cross-sectional analysis and a one-dimensional, one-way coupled, thermoelastic beam analysis. The present theory is implemented into the computer program, variational-asymptotic beam sectional analysis (VABS). Several examples are studied using the present theory and the results from VABS are compared with available analytical solutions and the three-dimensional analysis using the commercial finite element package ANSYS.

1. Introduction

Composite materials are widely used because of their excellent strength to weight and stiffness to weight ratio, flexibility of tailoring their properties through engineering the microstructures. This is especially true in aerospace industries, the making of military and civil aerospace systems, where weight saving is very important. Furthermore, the cost of composite materials has decreased due to the improvement of manufacturing technologies.

Many engineering structures made with composite materials have one dimension much larger than the other two and can be modeled as beams. The classical lamination theory (CLT), the first-order shear-deformation theory (FSDT) and some other higher-order theories [3,19,8] for composite beams under mechanical loading have been covered in detail by Reddy [20]. Within the last few decades, a tremendous research effort has been invested in this area, and various approximate models have been proposed [7,10,9,24,14]. Most of the models are based on ad hoc kinematic assumptions, such as the well-known Euler-Bernoulli assumptions, or other *apriori* assumed displacement field in terms of the cross-sectional coordinates.

The similar approach is also extended to deal with the thermal problem of composite beams. Structures made with composite materials are more sensitive and vulnerable to temperature change than their isotropic counterpart. The reason is that the thermal expansion coefficients of different constituents of the material are usually dramatically different from each other resulting in high stresses due to temperature changes from stress free environment. Usually, to avoid the fully-coupled thermoelasticity problem, researchers adopted the quasisteady theory of linear thermoelasticity, neglecting the temperature changes due to deformations. Under this situation, the thermal problem separates into two problem to be solved consecutively: the heat conduction problem to solve for the thermal field and the one-way coupled thermoelastic problem for the structure under a prescribed thermal field. Usually in the literature only very limited cases are considered so that one dimension of the cross-section, say the width, is neglected in the solution, or the thermal conductivity in one direction, say along the thickness, is set to be identical for all layers, see, for example, [18,16,25,22]. Besides this restriction, apriori assumptions are often used to express the temperature distribution and displacement distribution through the thickness [12,11,15,17,23,2]. It is noted that the thermal problem of general composite beams is investigated using a finite element semi-discretization approach [5,6].

Recently, Hodges and his co-workers applied the variational-asymptotic method (VAM) [1] to construct efficient high-fidelity models for composite structures [27,26]. In this approach, the one-dimensional (1D) beam model are asymptotically reduced from the original three-dimensional (3D) problem without invoking the apriori assumptions commonly used in the literature. Starting from a variational statement of the original 3D analysis, we can mathematically split the original 3D problem into a two-dimensional (2D) cross-sectional analysis and 1D global analysis. The focus of this paper is to extend this approach to deal with

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thermal problem of general composite beams, such as helicopter rotor blades, slender wings, wind turbine blades, or other slender structures made of composites.

2. Three-dimensional formulation

As sketched in Fig. 1, a beam can be represented by a reference line r measured by x_1, and a typical cross-section s with h as its characteristic dimension and described by cross-sectional Cartesian coordinates x_α (Here and throughout the paper, Greek indices assume values 2 and 3 while Latin indices assume 1, 2, and 3. Repeated indices are summed over their range except where explicitly indicated). At each point along r, an orthonormal triad $\mathbf{b}_i$ is introduced such that $\mathbf{b}_i$ is tangent to x_i.

2.1. Heat conduction analysis

The 3D steady heat conduction problem of the composite beam is governed by the variation of the following functional:

$$\Pi = \int_0^L \left[U_T - \langle QT \rangle - \oint_{\partial\Omega} T\,\bar{q}\,ds \right] dx_1 - \langle \bar{q}_e T \rangle|_{x_1=L} - \langle \bar{q}_e T \rangle|_{x_1=0} \tag{1}$$

where U_T is the thermal potential and the expression is

$$U_T = \left\langle \frac{1}{2} (\nabla T)^T K \nabla T \right\rangle \tag{2}$$

where angle bracket $\langle\ \rangle$ denotes integration over the cross-section, T is the 3D temperature field, K is the conductivity matrix representing the second-order conductivity tensor expressed in the triad $\mathbf{b}_i$, Q is the density of internal heat source, $\bar{q}$ is the given heat flux on the lateral boundary surfaces $\partial\Omega$, and $\bar{q}_e$ is the given heat flux on the end surfaces. For prismatic beams, the temperature gradient ∇T is

$$\nabla T = \begin{Bmatrix} T' \\ T_{,2} \\ T_{,3} \end{Bmatrix} \tag{3}$$

where $()' = \frac{\partial}{\partial x_1}$ and $()_{,\alpha} = \frac{\partial}{\partial x_\alpha}$.

If the temperature field T is not prescribed at any point over the cross-section except the end surfaces at $x_1 = 0$ and $x_1 = L$ (thermal load case 1, see Fig. 2), we are free to use the following change of variables for the 3D temperature field:

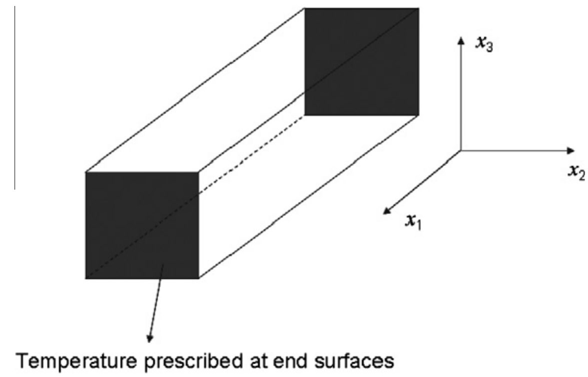

Fig. 2. Thermal load case 1: temperature prescribed at the end surfaces of the beam.

$$T(x_1, x_2, x_3) = \mathcal{T}(x_1) + w_T(x_1, x_2, x_3) \tag{4}$$

with the 1D temperature variable $\mathcal{T}(x_1)$ defined as the average of T over the cross-section, which implies

$$\langle w_T(x_1, x_2, x_3) \rangle = 0 \tag{5}$$

should be valid for all the surfaces along the span including the end surfaces. Note for thermal load case 1, it is possible that there is no temperature prescribed at one of the end surfaces or both end surfaces.

Here, we term the difference between the 3D temperature field and its cross-sectional average, w_T, as the thermal warping function. The temperature gradient can be expressed in the following form:

$$\nabla T = e_1 \mathcal{T}' + \Gamma_T w_T + e_1 w_T' \tag{6}$$

with $e_1 = \lfloor 1\ 0\ 0 \rfloor^T$, $\mathcal{T}'$ as 1D temperature gradient, and

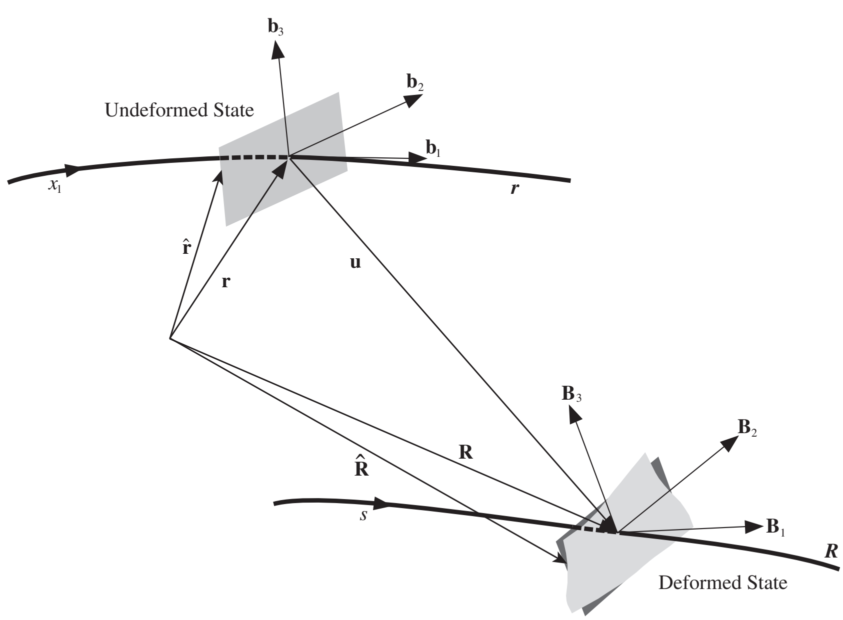

Fig. 1. Schematic of beam deformation.