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# A two variable refined plate theory for laminated composite plates

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ABSTRACT

A two variable refined plate theory of laminated composite plates is developed in this paper. The theory accounts for parabolic distribution of the transverse shear strains, and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factor. Equations of motion are derived from the Hamilton's principle. The closed-form solutions of antisymmetric cross-ply and angle-ply laminates are obtained using Navier solution. Numerical results of present theory are compared with three-dimensional elasticity solutions and results of the first-order and the other higher-order theories reported in the literature. It can be concluded that the proposed theory is accurate and simple in solving the static bending and buckling behaviors of laminated composite plates.

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## 1. Introduction

Fiber reinforced composite are widely used in the aerospace, automotive, marine and other structural applications. In the past three decades, researches on laminated composite plates have received great attention, and a variety of laminated theories has been introduced. The classical laminate plate theory (CLPT), which neglects the transverse shear effects, provides reasonable results for thin plates. However, the CLPT underpredicts deflections and overpredicts frequencies as well as buckling loads with moderately thick plates. Many shear deformation theories account for transverse shear effects have been developed to overcome the deficiencies of the CLPT. The first-order shear deformation theories (FSDTs) based on Reissner [1] and Mindlin [2] account for the transverse shear effects by the way of linear variation of in-plane displacements through the thickness. Since FSDT violates equilibrium conditions at the top and bottom faces of the plate, shear correction factors are required to rectify the unrealistic variation of the shear strain/stress through the thickness. In order to overcome the limitations of FSDT, higher-order shear deformation theories (HSDTs), which involve higher-order terms in Taylor's expansions of the displacements in the thickness coordinate, were developed by Librescu [3], Levinson [4], Bhimaraddi and Stevens [5], Reddy [6], Ren [7], Kant and Pandya [8], and Mohan et al. [9]. A good review of these theories for the analysis of laminated composite plates is available in Refs. [10–14]. A two variable refined plate theory (RPT) using only two unknown functions was developed by Shimpi [15] for isotropic plates, and was extended by Shimpi and Patel [16,17] for orthotropic plates. The most interesting feature of this theory is that it does not require shear correction factor, and has strong similarities with the classical plate theory in some aspects such as governing equation, boundary conditions and moment expressions.

The purpose of this paper is to develop the RPT for laminated composite plates. The present theory satisfies equilibrium conditions at the top and bottom faces of the plate without using shear correction factors. Governing equations are derived from the Hamilton's principle. Navier solution is used to obtain the closed-form solutions for simply supported antisymmetric cross-ply and angle-ply laminates. To illustrate the accuracy of the present theory, the obtained results are compared with three-dimensional elasticity solutions and results of the first-order and the other higher-order theories.

## 2. Refined plate theory for laminated composite plates

### 2.1. Basic assumptions

Consider a rectangular plate of total thickness $h$ composed of $n$ orthotropic layers with the coordinate system as shown in Fig. 1. Assumptions of the RPT are as follows:

(i) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.
(ii) The transverse displacement $W$ includes three components of extension $w_a$, bending $w_b$, and shear $w_s$. These components are functions of coordinates $x$, $y$, and time $t$ only.

$$W(x,y,z,t) = w_a(x,y,t) + w_b(x,y,t) + w_s(x,y,t) \tag{1}$$

(iii) The transverse normal stress $\sigma_z$ is negligible in comparison with in-plane stresses $\sigma_x$ and $\sigma_y$.

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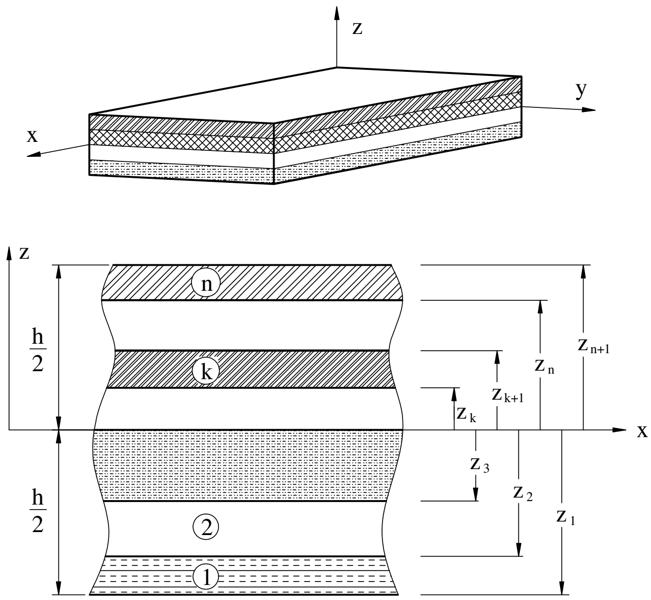


**Fig. 1.** Coordinate system and layer numbering used for a typical laminated plate.

(iv) The displacements $U$ in $x$-direction and $V$ in $y$-direction consist of extension, bending, and shear components.

$$U = u + u_b + u_s, \qquad V = v + v_b + v_s \tag{2}$$

– The bending components $u_b$ and $v_b$ are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for $u_b$ and $v_b$ can be given as

$$u_b = -z\frac{\partial w_b}{\partial x}, \qquad v_b = -z\frac{\partial w_b}{\partial y} \tag{3a}$$

– The shear components $u_s$ and $v_s$ give rise, in conjunction with $w_s$, to the parabolic variations of shear strains $\gamma_{xz}$, $\gamma_{yz}$ and hence to shear stresses $\sigma_{xz}$, $\sigma_{yz}$ through the thickness of the plate in such a way that shear stresses $\sigma_{xz}$, $\sigma_{yz}$ are zero at the top and bottom faces of the plate. Consequently, the expression for $u_s$ and $v_s$ can be given as

$$u_s = \left[\frac{1}{4}z - \frac{5}{3}z\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x}, \qquad v_s = \left[\frac{1}{4}z - \frac{5}{3}z\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y} \tag{3b}$$

### 2.2. Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1)–(3) as

$$\begin{aligned} U(x,y,z,t) &= u(x,y,t) - z\frac{\partial w_b}{\partial x} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \\ V(x,y,z,t) &= v(x,y,t) - z\frac{\partial w_b}{\partial y} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y} \\ W(x,y,z,t) &= w_a(x,y,t) + w_b(x,y,t) + w_s(x,y,t) \end{aligned} \tag{4}$$

The extension component $w_a$ of transverse displacement is negligible small in most cases. It can be neglected for a simpler version of the present theory named RPT1. The strains associated with the displacements in Eq. (4) are

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z\kappa_x^b + f\kappa_x^s \\ \varepsilon_y &= \varepsilon_y^0 + z\kappa_y^b + f\kappa_y^s \\ \gamma_{xy} &= \gamma_{xy}^0 + z\kappa_{xy}^b + f\kappa_{xy}^s \\ \gamma_{yz} &= \gamma_{yz}^a + g\gamma_{yz}^s \\ \gamma_{xz} &= \gamma_{xz}^a + g\gamma_{xz}^s \\ \varepsilon_z &= 0 \end{aligned} \tag{5}$$

where

$$\begin{aligned} &\varepsilon_x^0 = \frac{\partial u}{\partial x}, \quad \kappa_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad \kappa_x^s = -\frac{\partial^2 w_s}{\partial x^2} \\ &\varepsilon_y^0 = \frac{\partial v}{\partial y}, \quad \kappa_y^b = -\frac{\partial^2 w_b}{\partial y^2}, \quad \kappa_y^s = -\frac{\partial^2 w_s}{\partial y^2} \\ &\gamma_{xy}^0 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \kappa_{xy}^b = -2\frac{\partial^2 w_b}{\partial x \partial y}, \quad \kappa_{xy}^s = -2\frac{\partial^2 w_s}{\partial x \partial y} \\ &\gamma_{yz}^a = \frac{\partial w_a}{\partial y}, \quad \gamma_{yz}^s = \frac{\partial w_s}{\partial y} \\ &\gamma_{xz}^a = \frac{\partial w_a}{\partial x}, \quad \gamma_{xz}^s = \frac{\partial w_s}{\partial x} \\ &f = -\frac{1}{4}z + \frac{5}{3}z\left(\frac{z}{h}\right)^2, \quad g = \frac{5}{4} - 5\left(\frac{z}{h}\right)^2 \end{aligned} \tag{6}$$

### 2.3. Constitutive equations

Under the assumption that each layer possesses a plane of elastic symmetry parallel to the $x$–$y$ plane, the constitutive equations for a layer can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{7}$$

where $Q_{ij}$ are the plane stress-reduced stiffnesses, and are known in terms of the engineering constants in the material axes of the layer:

$$\begin{aligned} &Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \\ &Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \end{aligned} \tag{8}$$

Since the laminate is made of several orthotropic layers with their material axes oriented arbitrarily with respect to the laminate coordinates, the constitutive equations of each layer must be transformed to the laminate coordinates ($x$,$y$,$z$). The stress-strain relations in the laminate coordinates of the $k$th layer are given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)} \tag{9}$$

where $\overline{Q}_{ij}$are the transformed material constants given as

$$\begin{aligned} \overline{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\ \overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \\ \overline{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\ \overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta \\ \overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta \\ \overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta) \\ \overline{Q}_{44} &= Q_{44}\cos^2\theta + Q_{55}\sin^2\theta \\ \overline{Q}_{45} &= (Q_{55} - Q_{44})\cos\theta\sin\theta \\ \overline{Q}_{55} &= Q_{55}\cos^2\theta + Q_{44}\sin^2\theta \end{aligned} \tag{10}$$

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