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Energy release rates for interlaminar delamination in laminates considering transverse shear effects

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ABSTRACT

This paper presents simple and closed-form formulas for calculating mode I and II energy release rates, G_I and G_{II}, of a straight interlaminar crack in composite laminates subjected to general loadings. In addition to general combination of geometrical dimensions and material properties, the formulas are also explicitly expressed in terms of generic stress resultants in the cross section at the crack tip. The energy release rates predicted by the present analytical solutions are compared with those in the literatures. The novel formulations are presented and the simple closed-form solutions are obtained in terms of a general material and geometry combination and arbitrary loadings as well as the interface deformations.

1. Introduction

Layered structures are important and can be found in many applications, e.g., composite laminates, adhesively bonded joints, thermal barrier coatings, metallic thin film deposited on flexible substrate in electronics, and surface bonded ferroelectric (PZT or PLZT) wafers in smart structures. For a layered structure, delamination or interface fracture between two adjacent layers is a typical failure mechanism. Prediction of its failure needs to determine the total energy release rate and its mode ratio. Fig. 1 depicts schematically an important problem of a two-layered beam, for which one needs to calculate the mode I and II energy release rates for delamination or debonding of laminates with a general geometry and material combination and under axial loads, bending moments and transverse shear forces [1–3]. There are many attempts to solve this problem analytically by considering various geometrical and material as well as load combinations. For example, Gautesen and Dundurs [4] studied interface cracking for the case of tensile loading for a general geometrical configuration with bi-material substrates. Williams [1] derived the formulas for mode I and II energy release rates for the interface crack of layered structures subjected to bending moments, and then included the axial loads and shear forces by superposition. Suo and Hutchinson [3] derived stress intensity factors for the bi-material interface fracture subjected to axial loads and bending moments.

Further studies have been carried out to enhance the developed analytical solutions by taking into account the crack tip or interface deformations [5–7]. In an earlier work by Kanninen [8], the Winkler foundation was used to include interface deformation. Bruno et al. [9] employed a linear interface model between tractions and sliding/opening displacement jumps at the interface and included the transverse shear deformations by using the Reissner–Mindlin plates, and then calculated numerically the mixed-mode energy release rates by letting the interfacial stiffness approach infinity using an iterative collocation method in commercial software MATLAB. The results in [9] shows that it is important to include the shear forces and the interface deformation in the calculation of mode I and II energy release rates.

Due to difficulties encountered in the development of analytical solutions, finite element analysis (FEA) has been widely-used [10–13]. Li et al. [12] combined the energy release rates owing to the transverse shear forces calculated using FEA with those due to axial forces and bending moments predicted using the solutions of Suo and Hutchinson [3]. They believed that it did not seem to be possible to derive rigorous expressions for the shear force component of the energy release rate based on steady-state energy arguments or on any type of beam theory, as effects of shear forces depend on the details of the deformations in the crack tip region.

In this paper, we developed simple and closed-form solutions (see Eq. (4.12)) of mode I and II energy release rates for the general interface crack problem as shown in Fig. 1 by using the Timoshenko beam-interface model and the approach of letting the interfacial stiffness approach infinity as presented in [9]. The energy release rates are applicable to a general geometrical and material combination and are expressed in terms of all axial loads, bending moments and shear forces at the cross section of the crack tip. The present solutions are then compared with results available in the literatures. The present novel analytical solutions include the effects of transverse shear forces and interface deformations.

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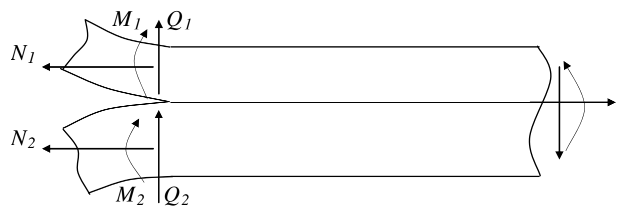

Fig. 1. Interlaminar delamination of a layer structure subjected to general stress resultants.

2. Adhesive interface model

2.1. Fundamental formulation

Consider the adhesive interface model shown in Fig. 2a. It is assumed that a straight crack propagate along midline of the adhesive layer that is sandwiched between two substrates of different materials and thickness. By referring to Fig. 2b, the equilibrium equations for substrates 1 and 2 are:

$$\begin{cases} \dfrac{dN_1}{dx}+\tau=0; \dfrac{dQ_1}{dx}+\sigma=0; \dfrac{dM_1}{dx}+\dfrac{t_1}{2}\tau-Q_1=0 \\ \dfrac{dN_2}{dx}-\tau=0; \dfrac{dQ_2}{dx}-\sigma=0; \dfrac{dM_2}{dx}+\dfrac{t_2}{2}\tau-Q_2=0 \end{cases} \tag{2.1}$$

in which all the stress resultants are defined in Fig. 2b; t_1 and t_2 are the thickness of substrates 1 and 2; τ and σ are the interface shear and peel stresses, which can be defined as [14]:

$$\tau = k_\tau(\Delta\bar{u}) = \frac{\overline{G}_a}{t_a}\left[(u_2-u_1)+\left(\frac{t_2}{2}\phi_2+\frac{t_1}{2}\phi_1\right)\right];$$
$$\sigma = k_\sigma(\Delta\bar{w}) = \frac{\overline{E}_a}{t_a}(w_2-w_1) \tag{2.2}$$

where $\Delta\bar{u}$ and $\Delta\bar{w}$ are interface displacement jumps between substrates 1 and 2; $k_\tau(=\overline{G}_a/t_a)$ and $k_\sigma(=\overline{E}_a/t_a)$ are the shear and peel stiffness; t_a is the adhesive thickness; $\overline{G}_a$ and $\overline{E}_a$ are the effective elastic moduli of the adhesive [15]. All other variables have the usual meanings used in the Timoshenko beam theory. Eq. (2.2) is a widely-used two-parameter linearly elastic interface model. The constitutive relations of the Timoshenko beam are:

$$N_i = A_{di}\frac{du_i}{dx}; \quad Q_i = G_{ki}\left(\frac{dw_i}{dx}-\phi_i\right); \quad M_i = -D_i\frac{d\phi_i}{dx} \qquad (i=1,2) \tag{2.3}$$

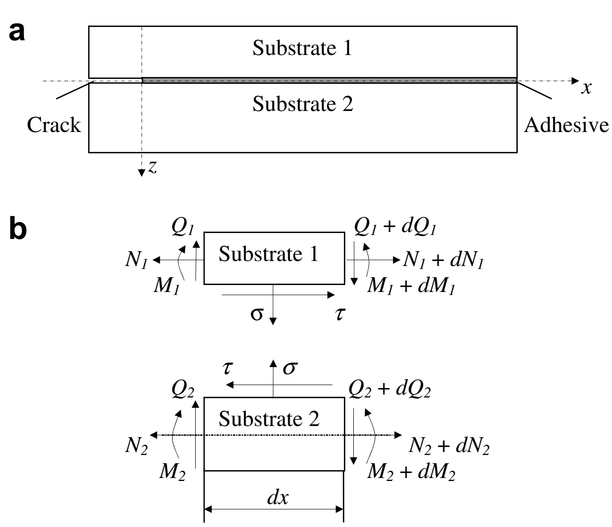

Fig. 2. (a) Crack in bondline; (b) free body diagrams of infinitesimal substrate elements.

where A_{di}, G_{ki} and D_i ($i = 1, 2$) are the extensional, shear and bending stiffness of substrates 1 and 2. When the substrates are composites, symmetrical lay-ups are assumed in Eq. (2.3).

To derive the governing equations, one needs to establish the following relations between the angle of rotations and curvatures by using Eqs. (2.1) and (2.3):

$$\frac{d\phi_1}{dx} = \frac{d^2w_1}{dx^2}+\frac{\sigma}{G_{k1}}; \frac{d\phi_2}{dx} = \frac{d^2w_2}{dx^2}-\frac{\sigma}{G_{k2}} \tag{2.4}$$

By substituting Eqs. (2.1), (2.3) and (2.4) into Eq. (2.2), one can obtain the following governing equations expressed in terms of shear and peel stresses:

$$\begin{cases} \dfrac{d^3\tau}{dx^3} = k_1\dfrac{d\tau}{dx}+k_2\sigma \\ \dfrac{d^4\sigma}{dx^4} = k_5\dfrac{d^2\sigma}{dx^2}-k_4\sigma-k_3\dfrac{d\tau}{dx} \end{cases} \tag{2.5}$$

where

$$\begin{cases} k_1 = k_\tau k_{1d}; \quad k_2 = k_\tau k_{2d}; \quad k_3 = k_\sigma k_{2d}; \quad k_4 = k_\sigma k_{4d}; \quad k_5 = k_\sigma k_{5d} \\ k_{1d} = \dfrac{4\alpha_{a1}}{A_{d1}}+\dfrac{4\alpha_{a2}}{A_{d2}}; \quad k_{2d} = \dfrac{t_1}{2D_1}-\dfrac{t_2}{2D_2}; \\ \quad k_{4d} = \dfrac{1}{D_1}+\dfrac{1}{D_2}; \quad k_{5d} = \dfrac{1}{G_{k1}}+\dfrac{1}{G_{k2}} \\ \alpha_{ai} = \dfrac{1+\alpha_{ki}}{4}; \alpha_{ki} = \dfrac{A_{di}t_i^2}{4D_i} \qquad (i=1,2) \end{cases} \tag{2.6}$$

In Eq. (2.6) k_{1d} and k_{4d} are the shear and peel coefficients of the interface; k_{5d} corresponds to the shear stiffness of the substrates; k_{2d} reflects the substrate asymmetry (for the symmetrical substrates $k_{2d} = 0$). The parameters α_{ai} and α_{ki} ($i = 1, 2$) vary with the lay-up sequences for composite laminates; and $\alpha_{ai} = 1$ and $\alpha_{ki} = 3$ ($i = 1, 2$) for isotropic substrates. Eq. (2.5) is a set of ordinary differential equations with both shear and peel stresses being coupled, and will be analytically solved for prescribed boundary conditions.

The governing differential equations shown in Eq. (2.5) are derived in light of the equilibrium equations in Eq. (2.1), interface stress definitions in Eq. (2.2) and the constitutive relations in Eq. (2.3). It is known that the transverse normal strains are not modeled in the Timoshenko beam theory. That is, material constant E_{33} is not included in Eq. (2.5). Kanninen [8] considered E_{33} by using the Winkler foundation. Szekrenyes and Uj [13] indicated that using the Winkler foundation or interface compliances to include E_{33} can improve predictions for energy release rates. Alternatively, effects of E_{33} on energy release rates can also be modeled using a higher order or layer-wise beam theory. In this paper, our purpose is to derive the analytical solutions based on the numerical model of Bruno et al. [9] without introducing other assumptions.

The boundary conditions for problem shown in Fig. 1 can be defined as:

$$x \to \infty: \quad \tau = 0; \quad \sigma = 0 \tag{2.7}$$

$$x = 0: \quad \frac{d\tau}{dx} = H_n; \quad \frac{d^2\sigma}{dx^2}-k_5\sigma = H_m;$$
$$\frac{d^3\sigma}{dx^3}-k_5\frac{d\sigma}{dx}+k_3\tau = H_q \tag{2.8}$$

where

$$\begin{cases} H_n = k_\tau H_{nd}; \quad H_m = k_\sigma H_{md}; \quad H_q = k_\sigma H_{qd} \\ H_{nd} = \left(\dfrac{N_2}{A_{d2}}-\dfrac{N_1}{A_{d1}}\right)-\frac{1}{2}\left(\dfrac{t_2M_2}{D_2}+\dfrac{t_1M_1}{D_1}\right); \quad H_{md} = -\left(\dfrac{M_2}{D_2}-\dfrac{M_1}{D_1}\right); \\ H_{qd} = -\left(\dfrac{Q_2}{D_2}-\dfrac{Q_1}{D_1}\right) \end{cases} \tag{2.9}$$

In Eqs. (2.7) and (2.8), a semi-infinitive length of the adhesive-bonded two-layered beam is assumed. In this paper, very thin