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Simple and efficient interlaminar stress analysis of composite laminates with internal ply-drop

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Abstract

A stress function-based variational method is developed to investigate the interlaminar stresses near the dropped plies. The present method is based on expanding stress functions in terms of harmonic series to the out-of-plane direction. The stresses satisfying the traction free boundary conditions of tapered laminates are obtained by the complementary strain energy principle. Stress concentrations near the dropped plies are predicted well as the number of initially assumed out-of-plane stress functions is increased. The proposed method is relatively simple and efficient in the prediction of interlaminar stresses near the dropped plies. Therefore, it is expected that the proposed method can be used in the parametric study of the preliminary design stage of tapered composite laminates.
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Keywords: Stress concentration; Interlaminar stress; Stress function; Internal ply-drop; Delamination fraction

1. Introduction

Stress singularity/concentration generally occurs at the dropped plies of composite laminates and can damage or failure of the laminates. Thus, the accurate prediction of interlaminar stresses is critical in the design of tapered laminates. Stress analysis near the dropped plies has been an important research topic for last decades. Stress concentrations near the dropped plies are similar to those of free-edge of laminated composites. Numerous researches have been conducted to investigate serious stress concentration/singularity near the free edges and near the dropped plies of composite laminates, which are caused by dissimilar material properties and geometries between plies [1–12]. Since the difficulties occur during the process of obtaining the exact singular elasticity solutions of interlaminar stresses, approximate methods based on numerical or analytical approaches have been pursued. Although recently developed numerical methods consist of either finite element methods or boundary integral methods, simple and accurate analytical methods are preferred in the preliminary design stage since they facilitate parametric study. After Pipes and Pagano proposed free-edge interlaminar stress problem [1], linear elastic models and simple regular stress function-based approximation methods for the analysis of interlaminar stresses have been studied. Present analysis method for tapered laminates is based on the stress function-based variational method of free edge interlaminar stresses proposed by Cho and Yoon [2] & Cho and Kim [3]. In the case of the presence of free edge at the boundary, the layup optimization considering uncertainty of material properties were conducted by Cho and Rhee [4,5].

Fish and Lee showed the delamination effects of interlaminar stresses of laminates with internal ply-drops [6]. To analyze interlaminar stresses, they used 2-D finite element method. Botting et al. reported the effects of ply-drop based on FEM and experimental results [7]. They used 3-D solid finite element to analyze interlaminar stresses. Mukherjee and Varughese proposed global-local scheme to reduce the number of degrees of freedom [8]. However, FEM approaches still require large amounts of computer

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Nomenclature

ε_i, σ_i	engineering strain and stress components in Cartesian coordinate system
S_{ij}	compliance in Cartesian coordinate system
α_i	coefficients of thermal expansion in Cartesian coordinate system
ΔT	temperature rising
F, Ψ	Lekhnitskii stress functions
f_i, p_i	in-plane stress functions
g_i	out-of-plane stress function
M	number of expansion terms in stress functions
N_{yy}, N_{xy}	longitudinal compression and in-plane shear
h	thickness of tapered laminates
γ_k	slope of each interface
N	total number of layers
η	non-dimensionalized coordinate in the z direction
z_t, z_b	functions of the top and bottom curves
U	complementary strain energy
t_d	thickness of dropped plies
F	delamination index
Z^{S1}, Z^{S2}, Z^T	allowable interlaminar stresses

resources. Harrison and Johnson proposed a mixed variational analytical formulation for reliable approximations of interlaminar stresses [9]. A mixed formulation has an advantage in describing displacement-prescribed boundary conditions and displacement continuity conditions at the interfaces between layers but it has too many primary variables. On the other hand, a stress function-based variational method shows simple and efficient approximation in the prediction of interlaminar stresses only with stress variables. The formulation of the present study is based on complementary strain energy principle with stress variables only. The objective of the present study is to provide simple analytical method to predict interlaminar stresses near the dropped plies of tapered composite laminates by the complementary energy approach. Longitudinal compression, in-plane shear, and transverse stretching are considered as mechanical loading. Uniform temperature change is considered as thermal loading. The proposed method can be used as a simple and efficient analytical tool in the prediction of stresses and strength of the laminates with internal ply-drop in the preliminary design stage.

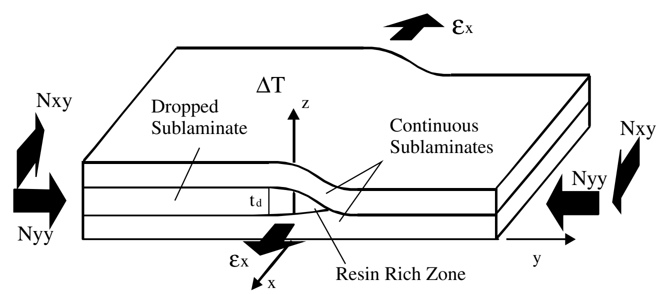

Fig. 1. Geometry of composite laminate with a single internal ply-drop.

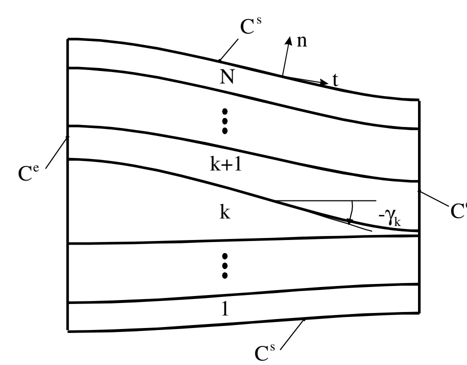

Fig. 2. Schematic view of tapered laminate.

2. Formulation

The geometry of composite laminates with internal ply-drop is given in Fig. 1. The laminate consists of orthotropic materials. The plies have arbitrary stacking angles with respect to the x-axis. The linear elastic constitutive equations are assumed in each ply and they are expressed as the following form.

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{Bmatrix}=\begin{bmatrix}S_{11}&S_{12}&S_{13}&0&0&S_{16}\\ S_{12}&S_{22}&S_{23}&0&0&S_{26}\\ S_{13}&S_{23}&S_{33}&0&0&S_{36}\\ 0&0&0&S_{44}&S_{45}&0\\ 0&0&0&S_{45}&S_{55}&0\\ S_{16}&S_{26}&S_{36}&0&0&S_{66}\end{bmatrix}\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{Bmatrix}+\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ 0\\ 0\\ \alpha_6\end{Bmatrix}\Delta T \tag{1}$$

where ε_i and σ_i represent engineering strain and stress components, respectively. The terms S_{ij} and α_i are compliance and thermal expansion coefficients of laminate and ΔT is temperature rising, respectively. From the first row of Eq. (1), following relationship can be obtained.

$$\sigma_1 = (\varepsilon_1 - \alpha_1\Delta T - S_{1j}\sigma_j)/S_{11} \quad j = 2,3,\ldots,6 \tag{2}$$

Substituting Eq. (2) into Eq. (1), all the strain components can be expressed as

$$\varepsilon_i = \widehat{S}_{ij}\sigma_j + \frac{S_{i1}}{S_{11}}\varepsilon_1 + \hat{\alpha}_i\Delta T \quad i,j = 2,3,\ldots,6 \tag{3}$$

where

$$\widehat{S}_{ij} = S_{ij} - \frac{S_{1i}S_{1j}}{S_{11}}, \quad \hat{\alpha}_i = \alpha_i - \frac{S_{1i}}{S_{11}}\alpha_1 \tag{4}$$

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