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Spectral element model for axially loaded bending–shear–torsion coupled composite Timoshenko beams

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ABSTRACT

The use of frequency-dependent spectral element matrix (or exact dynamic stiffness matrix) in structural dynamics is known to provide extremely accurate solutions, while reducing the total number of degrees-of-freedom to resolve the computational and cost problems. Thus, in this paper, the spectral element model is developed for an axially loaded bending–shear–torsion coupled composite laminated beam which is represented by the Timoshenko beam model based on the first-order shear deformation theory. The high accuracy of the spectral element model is then numerically verified by comparing with exact theoretical solutions or the solutions obtained by conventional finite element method. For the numerical verification, the finite element model is also provided for the composite laminated beam.

1. Introduction

It has been well recognized that fiber reinforced composite materials have many advantages over the isotropic materials due to their high strength-to-density ratios. Thus they have been widely used in many industrial applications [1–13]. Typical examples of industrial application are the composite aircraft wing, helicopter blade, propeller blade, turbine blade, axles of vehicle, thin-walled structures and so on. In general, the composite laminated structures are fabricated by bonding two or more laminae together. It is well known that the composite materials display anisotropic behavior and thus exhibit coupling between structural deformations. Thus the dynamic characteristics of a typical composite laminated structure can be enhanced by properly tailoring the ply orientation and stacking sequence.

Of particular interest especially for the high aspect ratio composite laminated beams (simply, composite beams) is the coupling between the bending and torsional modes of deformation, which is usually prevalent in aircraft wings or helicopter blades. As fibrous composite materials have usually very low shear modulus, the shear deformation should be also included in the theory of composite beams: this is true especially for the thin-walled composite structures. For the rotating composite beam structures such as helicopter blades, the effect of axial loading will be significant. Thus, this paper considers the dynamics of axially loaded bending–shear–torsion coupled composite beams, based on the first-order shear deformation theory (FSDT) which is the equivalent of the Timoshenko beam theory for composite beams.

Historically there have been many studies on the bending–torsional vibrations of composite beams. Among many researchers, Dukumaci [1], Hashemi and Richard [2] and Banerjee [3] have analyzed the dynamics of composite beams in the framework of the Bernoulli–Euler beam theory. Banerjee and Williams [4,5] considered the effects of shear deformation and rotary inertia by adopting the Timoshenko beam model based on FSDT. However they neglected the bending–torsional coupling from their Timoshenko beam model. The effect of bending–torsional coupling was included in the Timoshenko beam models by Teoh and Huang [6], Teh and Huang [7], Weisshaar and Foist [8], Banerjee [9,10], Li et al. [11] and Mei [12]. In the Timoshenko beam model by Banerjee and Williams [5], the elastic axis (the loci of the shear center of cross-section) was assumed to be deviated from the mass axis (the loci of the mass center of cross-section). On the other hand, in the Timoshenko beam models by Teoh and Huang [6], Teh and Huang [7], Banerjee [9,10], Li et al. [11] and Mei [12], the elastic axis was assumed to coincide with the mass axis. In this paper, the composite beams are represented by the Timoshenko beam models which include the bending–torsional coupling, axial load and damping effects. The elastic axis of the Timoshenko beam model will be considered to be deviated from the mass axis.

In the literature, various solution methods have been applied to the composite beams: analytical approaches [1,6,7,10], symbolic computing method [3], dynamic stiffness method [2,5,9,11], finite element method [8], mode superposition method [11], wave-train closure principle [12], and Rayleigh–Ritz method [13].

In the literature, the fast Fourier transforms (FFT)-based dynamic stiffness matrix method is often named spectral element method (SEM) [14]. Because the exact dynamic stiffness matrix is formulated from the exact frequency-dependent (dynamic) shape

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functions which satisfy the governing equations of motion, it represents the dynamic behavior of a structural element exactly. Thus, the SEM is often justifiably referred to as an exact solution method in the literature [15,16]. Accordingly, in contrast with the conventional finite element method (FEM), the SEM enables us to represent a whole uniform structural member as a single element, regardless of its length, without need to dividing the structural member into many fine elements in order to improve the solution accuracy. This will reduce the total number of degrees-of-freedom (DOFs) used in the dynamic analysis significantly, with lowering the computation cost. In the literature, Roy Mahapatra and Gopalakrishinan [17] and Gopalakrishinan et al. [18] applied the SEM to the axial–flexural–shear coupled wave propagation in composite beams.

Motivated from the aforementioned advantages of SEM, this paper presents a spectral element model for the axially loaded bending–shear–torsion coupled composite Timoskenko beams. The spectral element model developed for composite beams can be also applied to the dynamics of bending–shear–torsion coupled beam-type thin-walled structures. The accuracy of the present spectral element model is numerically evaluated by comparing the solutions obtained by using the present spectral element model with those obtained by using the conventional finite element model or analytical theories.

2. Governing differential equations of motion

The governing differential equations of motion and the associated boundary conditions for composite beams are derived by using Hamilton's principle given by

$$\int_0^t (\delta K - \delta V + \delta W)dt = 0 \tag{1}$$

where K is the kinetic energy, V is the strain energy, and δW is the virtual work done by external forces.

Consider a straight composite beam which takes a small amplitude bending–shear–torsion coupled vibration. The composite beam has the length L, the width b, and the thickness h, as shown in Fig. 1. The x-axis is chosen to coincide with the elastic axis which passes through the shear center of the beam cross-section. In Fig. 1, the coordinates system (1,2,3) represents the material coordinates for a specific laminate ply, which is rotated about z-axis (or 3-axis) by an specific ply orientation angle with respect to the inertial reference coordinates system (x,y,z). The displacements of the beam in the x-direction, y-direction and z-direction are represented by $u(x,y,z,t)$, $v(x,y,z,t)$ and $w(x,y,z,t)$, respectively. The elastic axis of the beam is assumed to take bending deflection $w_o(x,t)$ in the z-direction and torsional rotation $\phi(x,t)$ about the x-axis. The angle of rotation of the cross-section about the y-axis due to pure bending is represented by $\theta(x,t)$. Accordingly total slope w_o' equals the sum of slopes due to pure bending and shear deformation based on the FSDT which is the equivalent of the Timoshenko beam theory for composite beams. The displacements of the composite beam can be assumed in the approximated forms as

$$\begin{aligned} w(x,y,z,t) &\cong w_o(x,t) + y\phi(x,t) \\ u(x,y,z,t) &\cong -z\theta(x,t) \\ v(x,y,z,t) &\cong -z\phi(x,t) \end{aligned} \tag{2}$$

Assuming that the chordwise moment and the in-plane axial displacement are all negligible or zero, the resultant bending moment $M(x,t)$ and torque $T(x,t)$ can be related to the slope $\theta(x,t)$ and twist $\phi(x,t)$ as follows [8,9]:

$$\begin{Bmatrix} M \\ T \end{Bmatrix} = \begin{bmatrix} EI & C_{BT} \\ C_{BT} & GJ \end{bmatrix} \begin{Bmatrix} \theta' \\ \phi' \end{Bmatrix} \tag{3}$$

where EI, GJ, and C_{BT} are apparent bending rigidity, torsional rigidity, and bending–torsion material coupling rigidity, respectively, and they are defined by

$$\begin{aligned} EI &= b(D_{11} - D_{12}^2 D_{22}^{-1}) \\ GJ &= 4b(D_{66} - D_{26}^2 D_{22}^{-1}) \\ C_{BT} &= 2b(D_{16} - D_{12}D_{26}D_{22}^{-1}) \end{aligned} \tag{4}$$

where D_{ij} are the bending stiffnesses which are the functions of composite material properties, laminate ply orientation and stacking sequence [19]. Similarly, the resultant transverse shear force $Q(x,t)$ can be related to the bending deflection $w_o(x,t)$ and slope $\theta(x,t)$ as

$$Q(x,t) \cong \kappa GA(w_o' - \theta) \tag{5}$$

where κGA is the apparent shear rigidity defined by

$$\kappa GA = bA_{55} \tag{6}$$

where A_{55} is the extensional stiffness [19] and κ is the shear correction factor for the cross-section of the composite beam.

Assume that the composite beam is subjected to the forces $Q_1(t)$ and $Q_2(t)$, the moments $M_1(t)$ and $M_2(t)$, and the torques $T_1(t)$ and $T_2(t)$ applied at the boundaries $x = 0$ and $x = L$ as shown in Fig. 2, the distributed dynamic loads $f_w(x,t)$, $f_\theta(x,t)$ and $f_\phi(x,t)$, and a constant axial tensile force P which acts in the x-direction through the mass center of cross-section. By using the kinematic relations of Eq. (2) and the force–displacement relationships of Eqs. (3) and (5), the strain energy V, kinetic energy K, and the virtual work δW done by external forces can be obtained as follows:

$$\begin{aligned} V &= \frac{1}{2}\int_0^L \{M\theta' + T\phi' + Q(w_o' - \theta)\}dx + \frac{1}{2}\int_0^L \int_A Pw'^2\, dA\, dx \\ &= \frac{1}{2}\int_0^L \{EI\theta'^2 + 2C_{BT}\phi'\theta' + GJ\phi'^2 + \kappa GA(w_o' - \theta)^2 + P(w_o'^2 \\ &\quad + 2y_{CA}w_o'\phi' + r_g^2\phi'^2)\}dx \end{aligned}$$

$$\begin{aligned} K &= \frac{1}{2}\int_0^L \int_A \rho(\dot{w}^2 + \dot{u}^2 + \dot{v}^2)\, dA\, dx \\ &= \frac{1}{2}\int_0^L (\rho A\dot{w}_o^2 + 2\rho A y_{CM}\dot{w}_o\dot{\phi} + \rho I\dot{\theta}^2 + \rho J\dot{\phi}^2)\, dx \end{aligned} \tag{7}$$

$$\begin{aligned} \delta W &= Q_1\delta w_o(0) + Q_2\delta w_o(L) + M_1\delta\theta(0) + M_2\delta\theta(L) + T_1\delta\phi(0) \\ &\quad + T_2\delta\phi(L) + \int_0^L \{f_w(x,t)\delta w_o + f_\theta(x,t)\delta\theta + f_\phi(x,t)\delta\phi \\ &\quad - c_1\dot{w}_o\delta w_o - c_2\dot{\theta}\delta\theta - c_3\dot{\phi}\delta\phi\}\, dx \end{aligned}$$

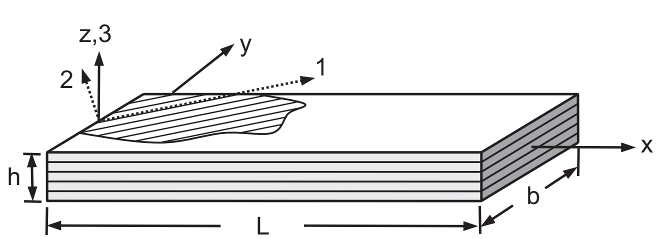

Fig. 1. The geometry and coordinates for a composite beam.

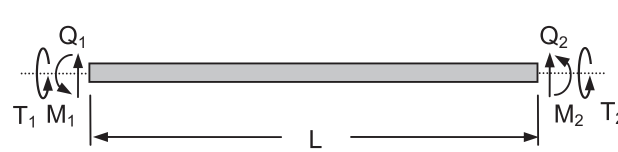

Fig. 2. Boundary forces for a composite beam.