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Axisymmetric bending and buckling analysis of thick functionally graded circular plates using unconstrained third-order shear deformation plate theory

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ABSTRACT

In the present article, axisymmetric bending and buckling of perfect functionally graded solid circular plates are studied based on the unconstrained third-order shear deformation plate theory (UTST). The UTST releases the shear-free condition on the top and bottom surfaces of plate which can be particularly useful when the plate is subjected to contact friction or presented in a flow field where the boundary layer shear stress is substantial. The solutions for deflections, force and moment resultants and critical buckling loads in bending and bucking analysis of functionally graded circular plates using UTST are presented in terms of the corresponding quantities of the homogeneous plates based on the classical plate theory (CPT). It is assumed that the non-homogeneous mechanical properties of plate, graded through the thickness, are described by a power function of the thickness coordinate. Resulting equations are employed to obtain the closed-form solutions. Numerical results for the maximum displacement and critical buckling load are presented for various percentages of ceramic-metal volume fractions and have been compared with those obtained using first- and third-order shear deformation plate theories.

1. Introduction

Functionally graded materials (FGMs) have gained much attention as advanced structural materials in recent years because of their heat-resistance properties. FGMs were first introduced in 1984 by a group of material scientists in Japan for developing thermal barrier materials [1,2]. In an FGM, the composition and structure gradually vary over volume, resulting in corresponding changes in the properties of the material. FGMs are made by combining two or more materials using powder metallurgy method. Typically these materials are made from a mixture of ceramic and metal in which the ceramic component provides high-temperature resistance due to its low thermal conductivity; on the other hand, the ductile metal component prevents fracture caused by thermal stresses. Functionally graded materials are now considered as a potential structural material for high-speed spacecraft. Several studies have been performed to analyze the behavior of functionally graded plates. Elastic bifurcation buckling of functionally graded plates under in-plane compressive loading was studied by Feldman and Aboudi [3], based on a combination of micromechanical and structural approaches. Javaheri and Eslami [4,5] reported mechanical and thermal buckling of rectangular functionally graded plates based on the classical Kirchhoff plate theory. Motivated by Javaheri, Lanhe [6] used Navier solution to study thermal buckling of a simply supported moderately thick rectangular FGM plate based on the first-order shear deformation plate theory (FST), and presented the results in closed-form solutions. Ng et al. [7] presented a formulation for the parametric response of functionally graded cylindrical shells under harmonic axial loading. They used Bolton's first approximation and considered the effects of the volume fraction of the material constituents and their distribution on the dynamic stability. By using an asymptotic method, Reddy and Cheng [8] studied three-dimensional thermomechanical deformations of simply supported, functionally rectangular plates. They computed temperature, displacements, and stresses of the plate for different volume fractions of the ceramic and metallic constituents. Praveen and Reddy [9] investigated the static and dynamic responses of functionally graded ceramic-metal plates by using a plate finite element that accounts for the transverse shear strains. Loy et al. [10] presented a study on the vibration of cylindrical shells made of FGM composed of stainless steel and nickel. They considered the influence of constituent volume fractions and the effects of configurations of the constituent materials on the frequencies. Using the first-order shear deformation plate theory of Mindlin, Reddy et al. [11] studied axisymmetric bending and stretching of functionally graded solid circular and annular plates. They presented the solutions for deflections, force and moment resultants based on the first-order plate theory in terms of those obtained using the classical plate theory. As an extensive work, Ma and Wang [12] employed TST to solve the

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bending and buckling problems of functionally graded materials. They derived the relationships between the solutions of axisymmetric bending and buckling of FGMs based on TST and those of homogeneous plates based on the classical Kirchhoff plate theory. Also, using shooting method, they considered the axisymmetric large deflection bending and thermal post-buckling behavior of functionally graded circular plates, under mechanical and thermal loadings based on the classical nonlinear von Karman plate theory [13]. Using the exact element method, Efraim and Eisenberger [14] analyzed the vibration of variable thickness thick annular homogeneous and FGM plates based on the first-order shear deformation theory. In the present work, unconstrained third-order shear deformation theory is employed to analyze the axisymmetric bending and buckling problems of functionally graded circular plates in which the bending-stretching coupling exists. Using an analytical method, the solutions for maximum displacements and critical buckling loads of functionally graded plates was derived based on the unconstrained-order shear deformation plate theory. The solutions have been derived in terms of the responses of homogeneous circular plates based on the classical plate theory. The effects of the material distribution through the thickness and shear deformation on the axisymmetric bending and buckling of the functionally graded plates have been considered.

2. Material properties

Consider a solid FGM circular thick flat plate of radius b and thickness h referred to the polar coordinate (r, θ, z). The materials in top and bottom surfaces of the plate are metal and ceramic, respectively. The material properties P of FGMs are a function of the material properties and volume fractions of the all constituent materials which can be expressed for k different constituents as

$$P = \sum_{i=1}^{k} P_i V_i, \tag{1}$$

where P_i and V_i are respectively, the material property and volume fraction of the ith constituent material. The sum of volume fractions of the all constituent materials must be unity as follow [10]

$$\sum_{i=1}^{k} V_i = 1, \tag{2}$$

It is assumed that the material composition in an FGM plate varies continuously along the thickness direction only, in terms of volume fractions according to a power law distribution as

$$V_{\mathrm{m}}(z) = \left(\frac{h-2z}{2h}\right)^N, \tag{3}$$

where subscript m refers to the metal component; and N denotes the power law index which takes values greater than or equal to zero. The value of N equal to zero represents the fully ceramic plate.

For a FGM with two constituent materials, the volume fraction of ceramic component can be obtained in view of the Eqs. (2) and (3) as follow

$$V_{\mathrm{c}} = 1 - \left(\frac{h-2z}{2h}\right)^N, \tag{4}$$

where subscript c refers to the ceramic component.

Using Eqs. (1), (3) and (4), the Young's modulus E and Poisson ratio ν can be written as

$$E = (E_{\mathrm{m}} - E_{\mathrm{c}})\left(\frac{h-2z}{2h}\right)^N + E_{\mathrm{c}}, \tag{5a}$$

$$\nu = (\nu_{\mathrm{m}} - \nu_{\mathrm{c}})\left(\frac{h-2z}{2h}\right)^N + \nu_{\mathrm{c}}. \tag{5b}$$

Generally Poisson's ratio ν varies in a small range, for simplicity it is assumed to be a constant.

3. Formulation of the problem

3.1. Equilibrium equations based on UTST

The unconstrained third-order shear deformation theory is based on the following representation of the displacement field across the plate thickness [15]

$$U_{\mathrm{r}}(r,z) = u(r) + z\varphi_1(r) + z^3\varphi_2(r), \tag{6a}$$

$$U_{\mathrm{z}}(r,z) = w(r), \tag{6b}$$

where U_{r} and U_z are the displacements along the radius and thickness, respectively; u is the stretching function; φ_1 is the rotation function; φ_2 is the higher-order rotation function; and w is the transverse deflection. The linear strain components associated with the displacement field (6) are

$$\varepsilon_{\mathrm{rr}} = \frac{\partial U_{\mathrm{r}}}{\partial r} = \frac{\mathrm{d}u}{\mathrm{d}r} + z\frac{\mathrm{d}\varphi_1}{\mathrm{d}r} + z^3\frac{\mathrm{d}\varphi_2}{\mathrm{d}r}, \tag{7a}$$

$$\varepsilon_{\theta\theta} = \frac{U_{\mathrm{r}}}{r} = \frac{u}{r} + z\frac{\varphi_1}{r} + z^3\frac{\varphi_2}{r}, \tag{7b}$$

$$\gamma_{\mathrm{rz}} = \frac{\partial U_{\mathrm{r}}}{\partial z} + \frac{\partial U_{\mathrm{z}}}{\partial r} = \varphi_1 + \frac{\mathrm{d}w}{\mathrm{d}r} + 3z^2\varphi_2. \tag{7c}$$

The Hooke's law for a plate is defined as

$$\sigma_{\mathrm{rr}} = \frac{E}{1-\nu^2}(\varepsilon_{\mathrm{rr}} + \nu\varepsilon_{\theta\theta}), \tag{8a}$$

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}(\varepsilon_{\theta\theta} + \nu\varepsilon_{\mathrm{rr}}), \tag{8b}$$

$$\sigma_{r\theta} = \frac{E}{2(1+\nu)}\varepsilon_{r\theta}. \tag{8c}$$

Using the principle of virtual displacements, the equilibrium equations can be obtained as [16]

$$\delta u: \quad \frac{\mathrm{d}}{\mathrm{d}r}(rN_{\mathrm{rr}}) - N_{\theta\theta} = 0, \tag{9a}$$

$$\delta\varphi_1: \quad \frac{\mathrm{d}}{\mathrm{d}r}(rM_{\mathrm{rr}}) - M_{\theta\theta} - rQ_{\mathrm{r}} = 0, \tag{9b}$$

$$\delta\varphi_2: \quad \frac{\mathrm{d}}{\mathrm{d}r}(rP_{\mathrm{rr}}) - P_{\theta\theta} - 3rR_{\mathrm{r}} = 0, \tag{9c}$$

$$\delta w: \quad \frac{\mathrm{d}}{\mathrm{d}r}(rQ_{\mathrm{r}}) + rq = 0. \tag{9d}$$

where δ represents the variational symbol; N_i, M_i, P_i, (i = rr, $\theta\theta$) are the forces, moments, and higher-order moments, respectively; and Q_{r}, R_{r} are respectively, shear force and higher-order shear force which are all defined by the following expressions

$$(N_{\mathrm{rr}}, M_{\mathrm{rr}}, P_{\mathrm{rr}}) = \int_{-h/2}^{h/2} \sigma_{\mathrm{rr}}(1, z, z^3)\mathrm{d}z, \tag{10a}$$

$$(N_{\theta\theta}, M_{\theta\theta}, P_{\theta\theta}) = \int_{-h/2}^{h/2} \sigma_{\theta\theta}(1, z, z^3)\mathrm{d}z, \tag{10b}$$

$$(Q_{\mathrm{r}}, R_{\mathrm{r}}) = \int_{-h/2}^{h/2} \sigma_{\mathrm{rz}}(1, z^2)\mathrm{d}z. \tag{10c}$$

Using Eqs. (7), (8), and (10), one can obtain the constitutive relations as

$$N_{\mathrm{rr}} = A_{11}\Lambda u + B_{11}\Lambda\varphi_1 + E_{11}\Lambda\varphi_2, \tag{11a}$$

$$N_{\theta\theta} = A_{11}\forall u + B_{11}\forall\varphi_1 + E_{11}\forall\varphi_2, \tag{11b}$$

$$M_{\mathrm{rr}} = B_{11}\Lambda u + D_{11}\Lambda\varphi_1 + F_{11}\Lambda\varphi_2, \tag{12a}$$

$$M_{\theta\theta} = B_{11}\forall u + D_{11}\forall\varphi_1 + F_{11}\forall\varphi_2, \tag{12b}$$

$$P_{\mathrm{rr}} = E_{11}\Lambda u + F_{11}\Lambda\varphi_1 + H_{11}\Lambda\varphi_2, \tag{13a}$$