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Buckling and vibration analysis of functionally graded composite structures using the finite element method

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ABSTRACT

The authors [Oyekoya OO, Mba DU, El-Zafrany AM. Structural integrity of functionally graded composite structure using Mindlin-type finite elements. ICCES 2008;172(1): 1–6] have previously written a paper on structural integrity of functionally graded composite (FGC) structure using Mindlin-type finite elements. In this paper, the Mindlin-type element and Reissner-type element have been further developed for the modelling of FGC plate subjected to buckling and free vibration. The Mindlin-type element formulation is based on averaging of transverse shear distribution over plate thickness using Lagrangian interpolation. The Reissner-type element formulation is based on parabolic transverse shear distribution over plate thickness using Lagrangian and Hermitian interpolation. The composite plate considered in this paper is functionally graded in the longitudinal direction only, but the FE code developed is capable of analysing composite plates with functional gradation in transverse and radial direction as well. This study was able to show that the structural integrity enhancement and strength maximisation of composite structures are achievable through functional gradation of material properties over the structure.

1. Introduction

Functionally graded materials (FGMs) are made such that the volume fractions of two or more materials are varied continuously along a certain dimension. The FGM concept originated in Japan in 1984 during the space-plane project, in the form of a proposed thermal barrier material capable of withstanding a surface temperature of 2000 K and a temperature gradient of 1000 K across a cross section <10 mm [2]. In 2000, Reddy [3] presented a theoretical formulation and finite element models based on third-order shear deformation theory for the analysis of through-thickness functionally graded plates. The Navier solution for simply supported plates based on the linear third-order theory and the non-linear static and dynamic finite element results based on the first-order theory were presented by Reddy in [3]. The results show the effects of volume fractions and modulus ratio of the constituents on deflections and transverse shear stresses. In 2002, Javaheri et al. [4] derived equilibrium and stability equations for rectangular simply supported functionally graded plates. Javaheri's derivation was based on the classical plate theory with the assumption of power law composition for the material and he studied the buckling analysis of functionally graded plates under in-plane compression. In 2004, Chen et al. [2] investigated the buckling behaviour of FGM rectangular plates subjected to non-linearly distributed in-plane edge loads. Chen et al. [2] stated that a mesh-free method which approximates displacements based on scattered nodes (i.e. radial basis function and polynomial basis) was employed, in order to avoid complicated numerical procedures that arises in the FEM from the use of elements. This FEM complication was dealt with in this paper. Other useful studies on functionally graded materials can be found in these Refs. [5–11].

In comparison to existing publications, this paper has been able to give unique contributions to the subject matter. These contributions include Mindlin-type element formulation, Reissner-type element formulation, finite strain modelling and smooth fibre distribution technique. The Mindlin-type element formulation is based on an assumption of average transverse shear distribution over plate thickness using Lagrangian interpolation. The Reissner-type element formulation is based on an assumption of parabolic transverse shear distribution over plate thickness using Lagrangian and Hermitian interpolation. Green's strain–displacement equation was employed in the finite strain modelling. The smooth fibre distribution technique is based on the numerical computation of macro-mechanical properties at Gaussian quadrature points.

The authors [1] have previously written a paper on structural integrity of functionally graded composite structure using Mindlin-type finite elements. In the paper [1], two new Mindlin-type plate bending elements were derived for the modelling of functionally graded plate subjected to various loading conditions such as tensile loading, in-plane bending and out-of-plane bending. There

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Nomenclature

$\mathbf{B}$	matrix of shape function derivatives
$\mathbf{F}$	nodal load vector
$\mathbf{K}$	element stiffness matrix
$\mathbf{q}$	interpolated displacement component
$\mathbf{R}$	residual vector
δ	displacement component
$\boldsymbol{\gamma}$	transverse shear strain vector
$\boldsymbol{\sigma}$	x–y stress vector
$\boldsymbol{\tau}$	transverse shear stress vector
$\boldsymbol{\varepsilon}$	x–y strain vector
$\vec{a}$	acceleration vector
F_i, G_i	Hermitian shape functions
H_i, P_i	Hermitian shape functions
N_i	Lagrangian shape functions
U	strain energy
u, v, w	displacement components along the x, y and z directions respectively
V	volume fraction
W	work done by actual load
X	longitudinal strength
Y	transverse strength
ξ, η	non-dimensional x and y location
ρ	density
μ	shear modulus
ψ	transverse shear deformation
ν	Poisson ratio
$(\ )_1$	longitudinal direction of the material axis
$(\ )_2$	transverse direction of the material axis
$(\ )_\mathrm{c}$	compressive strength
$(\ )_\mathrm{f}$	fibre
$(\ )_\mathrm{m}$	matrix
$(\ )_\mathrm{t}$	tensile strength
$(\ )_x$	longitudinal direction of the local axis
$(\ )_y$	transverse direction of the local axis
$(\ )_\mathrm{comp}$	traditional composite
$(\ )_\mathrm{fgm}$	functionally graded material
$(\ )_b$	out-of-plane displacement component
$(\ )_{uv}$	u and v displacement components
$(\ )_w$	w displacement component
$(\ )_o$	midplane displacement component
$(\ )_\psi$	transverse shear component
$(\ )_\theta$	bending component
$(\ )^L$	Lth layer of composite
$(\ )^o$	midplane displacement component
$(\ )^\sigma$	non-linear terms

were two types of non-linearity considered in the modelling of the plate, which include finite strain and material degradation. In the Mindlin-type element formulation, the transverse shear strain is averaged over plate thickness. Its finite element derivation is based on Lagrangian interpolation.

In order to model the transverse shear strain more accurately, the Reissner-type element is derived in this paper, which is based on parabolic distribution of transverse shear strain over plate thickness. Its finite element derivation is complex because it is based on Lagrangian and Hermitian interpolations.

In this paper, the Mindlin-type element and Reissner-type element have been further developed for the modelling of functionally graded composite plate subjected to buckling and free vibration. Vibration and buckling analysis were then undertaken for different fibre distribution cases and the effects of fibre distribution were studied. Fibre distribution cases with maximum vibration frequency and maximum buckling loads were chosen as the optimum design.

1.1. Optimisation technique

The optimisation technique used in this paper can be described as a fail-safe design technique which involves the imposition of constraints to ensure that the physical limitations of materials or structural properties required for satisfactory performance are not exceeded. This optimisation technique involves changing the fibre distribution parameters and running the FE code for the given fibre distribution, checking to see if all constraints have been satisfied. The constraints that have been considered include buckling load constraints and natural frequency constraints. Fig. 1 is a good description of this optimisation technique concept.

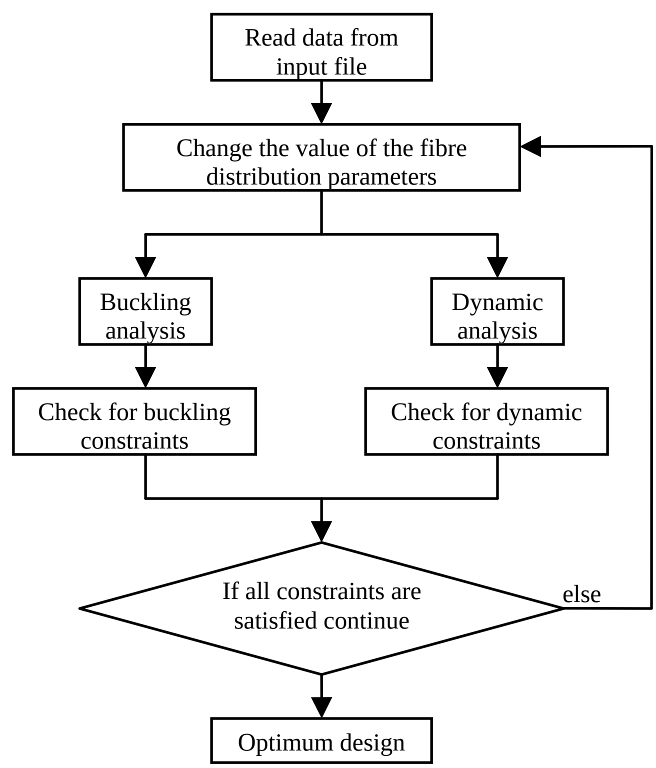

Fig. 1. Optimisation technique.

2. Micromechanics of fibrous composites

This section defines the elastic and strength properties of FGMs [12]. It also describes the micromechanics algorithm and the fibre distribution techniques such as average and smooth fibre distribution technique.

2.1. Elastic properties of FGMs

The longitudinal stiffness of a composite can be obtained from the rule or law of mixture which is represented by the relationship given below

$$E_{11} = E_\mathrm{f}V_\mathrm{f} + E_\mathrm{m}V_\mathrm{m} \qquad (1)$$